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# A novel parametric correlation coefficient for intuitionistic fuzzy numbers

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**Abstract.** Correlation coefficients are widely used to measure the relationship between variables, but existing approaches for intuitionistic fuzzy numbers (IFNs) often fail to capture negative dependence and lack flexibility. In this paper, we propose a novel parametric correlation coefficient for IFNs based on a newly defined difference operator. The proposed method allows the correlation value to vary within the interval $[-1, 1]$, enabling the representation of both positive and negative relationships. Moreover, by introducing parameters $\alpha$ and $\beta$, the method provides flexibility for decision-makers to evaluate correlations at different levels. Numerical examples and applications in medical diagnosis and multi-criteria decision-making demonstrate the effectiveness, robustness, and practical applicability of the proposed approach.

**Keywords.** Intuitionistic fuzzy correlation, Difference operator, Multi-criteria, Fuzzy number.

**MSC:** 62-XX, 62H20, 03E72.

## 1 Introduction

In 1885 and 1888, the hypothesis of two-variable normal correlation and the relationship between two variables was proposed for the first time by Galton. In 1895, Pearson presented a formula for calculating the correlation between two variables, which is used today as Pearson's correlation coefficient. Before the work of Galton and Pearson, relationships between variables were mainly explained through causal interpretations.

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Today, the correlation coefficient and the regression equation related to it cause the formation of basic statistical methods for observations and random variables in many mathematical structures. The correlation coefficient is a statistical index that measures the strength and direction of the link between two variables. Various methods are offered to interpret the correlation coefficient, for example, standardized covariance, standardized slope of the regression line, square root of the ratio of two variances, a function of the angle between two variable vectors, etc. It is claimed that there is a correlation between two variables if the values of both variables fluctuate in the same way, that is, with the increase or decrease of one, the other also decreases or increases, and this relationship may be written as an equation. The notion of correlation coefficient is widely used in the scientific and technical communities. In correlation analysis, a measure of the interdependence of the two variables is used to examine the joint relationship of the variables. One way to do this is by use a correlation coefficient. In statistics, the correlation coefficient takes on values between $-1$ and 1, with a value of 1 indicating a perfect linear relationship between the two variables being studied (i.e., an increase or decrease in both). However, the value of the correlation coefficient is $-1$ when two variables are linearly inverted. This indicates that when one variable increases in value, the other decreases. Since uncertainty is often present in MCDM difficulties, the idea of correlation coefficient has been extended to incorporate intuitionistic fuzzy set setting to make its applications practical.

However, it is challenging to provide a novel approach when data is seen with equivocal information. Therefore, fuzzy theory and other similar soft approaches of data analysis are required. Research on fuzzy correlation analysis has produced various approaches; however, several challenges remain in terms of flexibility and interpretability. The fuzzy correlation coefficient is the subject of many academic works. Several investigations have been performed when the data does not fit the criteria for general fuzzy data or interval estimate. Measurement Liu and Kao (2002); Liu et al. (2016) and t-norms Hong (2006); Saneifard et al. (2012) were utilized by some. Some studies have considered alternative interpretations of fuzziness and correlation structures (Chaudhuri and Bhattacharya, 2001).

Some studies have developed fuzzy correlation coefficients using optimization-based approaches. For instance, Liu and Kao (2002) employed linear programming techniques to derive $\alpha-$cuts of the fuzzy correlation coefficient. Although such methods provide relatively precise results, they often involve high computational complexity, especially in multi-step problems

Another group of approaches is based on t-norm operations. For example, Hong (2006) and Kumar (2019) utilized weakest t-norm-based fuzzy arithmetic to define correlation coefficients. These methods are mathematically consistent; however, they may lack flexibility and are often limited in representing complex relationships.

Distance-based approaches have also been widely used in the literature. Liu et al. (2016) and Thao (2020) proposed correlation measures based on distance and deviation functions. These methods are relatively intuitive and easy to implement, but in most cases, their outputs are restricted to the interval $[0, 1]$, which limits their ability to capture negative correlations.

Other contributions include statistical and aggregation-based approaches such as those proposed by Park et al. (2009) and Ye (2010), which incorporate membership, non-membership, and hesitation degrees. While these methods provide useful insights, they generally lack parametric flexibility and do not allow the decision maker to adjust the level of analysis.

From the above discussion, it can be observed that existing methods suffer from several limitations, including restricted output ranges, lack of flexibility, and high computational cost. In particular, most correlation coefficients fail to represent negative relationships and do not provide adjustable parameters for decision-making. Motivated by these limitations, this paper proposes a new parametric correlation coefficient for intuitionistic fuzzy numbers. In this study, we specifically focus on intuitionistic fuzzy numbers, which provide a more expressive framework than classical fuzzy numbers by incorporating membership, non-membership, and hesitation degrees.

Many of the correlation coefficients defined for numbers and fuzzy sets are in the range of 0 and 1, while in fact it should be between 1 and $-1$ because two random variables may have an inverse correlation. In this article, we present a correlation coefficient for IFNs, which overcomes this limitation and secondly, it is parametric. In the sense that the decision maker can calculate the correlation coefficient of two data sets of fuzzy numbers at different levels of decision-making by choosing the appropriate $\alpha$ and $\beta$. In this way, two sets of data may be correlated at one level of decision-making but not at all at another level or their correlation is much lower.

The remainder of this paper is organized as follows. Section 2 reviews the necessary preliminaries on intuitionistic fuzzy numbers. Section 3 introduces the proposed parametric correlation coefficient. Sections 4 and 5 demonstrate its applicability through medical diagnosis and MCDM problems. Finally, Section 6 provides concluding remarks.

## 2 Fuzzy arithmetic

In this section, some necessary prerequisites related to fuzzy calculations are stated. Fuzzy subsets were introduced by Lotfi Zadeh in 1965 by publishing a paper (Zadeh, 1965). He described membership in these subsets as imprecise and believed that many of the sets we deal with in reality are of this type. This paper opened a new chapter for new research on collections and their applications such as decision making in ambiguous spaces, analysis of imprecise systems and approximate reasoning. Although the theory of fuzzy sets has successfully handled uncertainties caused by ambiguity or partial belonging to a set, it cannot handle all uncertainties that often exist in real life problems, especially problems that involve insufficient information. To address this issue, Atanassov (1986) introduced a generalization of fuzzy sets called Intuitionistic Fuzzy Sets (IFS) that can represent another dimension of the membership function. Before introducing the arithmetic operations and representations of intuitionistic fuzzy numbers, we first recall the formal definition of intuitionistic fuzzy sets.

**Definition 2.1.** Let $X$ be a universe of discourse, then an IFS $\tilde{A}$ is defined as:

$$\tilde{A} = \{x, \mu(x), \nu(x) | x \in X\}$$

where $\mu : X \to [0,1]$ and $\nu : X \to [0,1]$, with $0 \leq \mu(x) + \nu(x) \leq 1$. The functions $\mu(x)$ and $\nu(x)$ represent the membership function and non membership function of $\tilde{A}$, respectively. In the above definition, membership function $\mu(x)$ and non-membership function $\nu(x)$ are both fuzzy sets. Therefore, intuitionistic fuzzy sets are also called two-dimensional fuzzy sets. If we set $\pi(x) = 1 - \mu(x) - \nu(x)$ then $\pi(x)$ is called the hesitation degree of $\tilde{A}$

*Remark* 2.2. Classical fuzzy numbers can be viewed as a special case of intuitionistic fuzzy numbers when $\nu(x) = 1 - \mu(x)$. The concept of $\alpha$-cuts and $\beta$-cuts plays an important role in fuzzy arithmetic and computational implementations of intuitionistic fuzzy sets. These level sets transform fuzzy quantities into interval-valued forms, which simplify mathematical operations and numerical analysis.

**Definition 2.3.** The $\alpha$-cuts for an IFS $\tilde{A}$ is defined as follows (Liu and Kao, 2002; Watanabe and Imaizumi, 1999):

1. $\tilde{A}_\alpha = \{x | \mu(x) \geq \alpha\}$.
2. $\tilde{A}_\beta = \{x | \nu(x) \leq \beta\}$

To perform arithmetic manipulations on intuitionistic fuzzy numbers, suitable definitions of addition and multiplication are required. These operations extend classical fuzzy arithmetic by incorporating both membership and non-membership information. The use of $\alpha$-cuts and related level-set representations in fuzzy arithmetic and correlation analysis has been extensively studied in the literature (Liu and Kao, 2002; Watanabe and Imaizumi, 1999). In the theory of intuitionistic fuzzy sets, operations of addition and multiplication are defined as follows.

**Definition 2.4.** Suppose $A$ and $B$ are two intuitionistic fuzzy sets. For each $u \in U$ the addition and multiplication operations for these two fuzzy numbers are defined as follows (Hong, 2006; Kumar, 2019; Ejegwa and Onyeke, 2021):

- $(A + B)(u) = (\mu_A(u) + \mu_B(u) - \mu_A(u).\mu_B(u), \nu_A(u)\nu_B(u))$
- $(A.B)(u) = (\mu_A(u)\mu_B(u), (\nu_A(u) + \nu_B(u) - \nu_A(u)\nu_B(u)))$

Among different types of intuitionistic fuzzy numbers, LR-type intuitionistic fuzzy numbers are particularly useful because of their flexible parametric representation and computational convenience. They are widely used in fuzzy decision-making and correlation analysis.

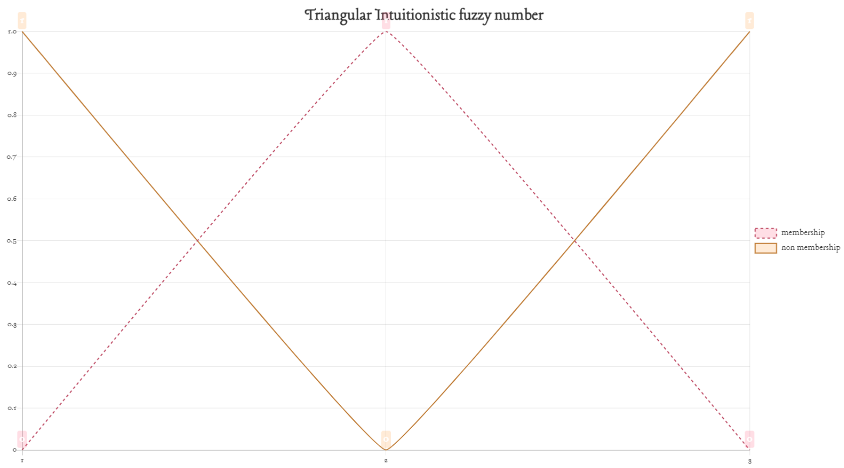


Figure 1: Triangular intuitionistic fuzzy number.

**Definition 2.5.** A LR-type IFN is defined as follows:

$$\mu(x) = \begin{cases} L(p^{-1}(m-x)), & x \leq m, \\ R(q^{-1}(x-m)), & x > m, \end{cases}$$

$$\nu(x) = \begin{cases} 1 - L(p'^{-1}(m-x)) & x \leq m, \\ 1 - R(q'^{-1}(x-m)) & x > m. \end{cases}$$

where $L$ and $R$ are strictly decreasing functions and $L(0) = R(0) = 1$. Also $p, p' \in \mathbf{R} \cap 0$ are called left spread and $q, q' \in \mathbf{R} \cap 0$ are called right spread. In particular case, if $L(x) = R(x) = [1-x]^+$, then $\tilde{A}$ is called a triangular IFN (TriIFN). Figure 1 shows a triangular intuitionistic fuzzy number whose the hesitation degree is zero.

LR-type intuitionistic fuzzy numbers are widely used due to their convenient parametric representation and computational efficiency. Several studies, such as Darehmiraki (2019) and Shakouri et al. (2020), have employed parametric forms of intuitionistic fuzzy numbers in ranking and decision-making models. For computational purposes, it is often more convenient to represent intuitionistic fuzzy numbers using functional or $r$-level representations. Such representations simplify analytical computations and facilitate the construction of distance and correlation measures. By expressing the membership and non-membership functions in terms of the $r$-level parameter, the following functional representation of intuitionistic fuzzy numbers is obtained.

**Definition 2.6.** Let $\tilde{A} = (\mu', \nu')$ be an intuitionistic fuzzy number represented in r-level form, where $\mu'$ is a pair $(\underline{m}(r), \overline{m}(r))$ of functions, $0 \leq r \leq 1$, which satisfies the following conditions (Shakouri et al., 2020):

1. $\underline{m}(r)$ is a bounded monotonic increasing left continuous function.

2. $\overline{m}(r)$ is a bounded monotonic decreasing left continuous function.

3. $\underline{m}(r) \leq \overline{m}(r)$, $0 \leq r \leq 1$.

$\nu^{'}$ is a pair $(\underline{n}(r), \overline{n}(r))$ of functions, $0 \leq r \leq 1$, which applies in the following conditions

1. $\underline{n}(r)$ is a bounded monotonic decreasing left continuous function.

2. $\overline{n}(r)$ is a bounded monotonic increasing left continuous function.

3. $\underline{n}(r) \leq \overline{n}(r)$, $0 \leq r \leq 1$.

$$\begin{cases} \mu^{'} = (m - p(1-r), m + q(1-r)), \\ \nu^{'} = (m - p^{'}(1-r), m + q^{'}(1-r)). \end{cases} \tag{2.1}$$

These foundational concepts form the basis for the development of the proposed correlation coefficient. To clarify the relationship between Definition 2.6 and Equation (2.1), note that the parameters $m, p, q, p^{'}, q^{'}$ in Equation (2.1) provide a parametric representation of the endpoint functions introduced in Definition 2.6. Specifically, $m$ denotes the central value, while $p$ and $q$ control the left and right spreads of the membership function. Similarly, $p^{'}$ and $q^{'}$ determine the spreads of the non-membership function. Therefore, the functions $\underline{m}(r)$ and $\overline{m}(r)$ correspond to the lower and upper bounds generated by these parameters under the $r$-level representation. This parametric form provides a more convenient computational representation of intuitionistic fuzzy numbers. Arithmetic operations on fuzzy and interval-valued fuzzy information have been extensively investigated in the literature. In this regard, the work of Deschrijver (2007) provides a comprehensive study of arithmetic operators for interval-valued fuzzy sets. Furthermore, Taheri and Zarei (2011) developed an extension principle for vague sets and discussed several applications.

## 3 Parametric fuzzy correlation coefficient

The Pearson correlation coefficient is a well-known measure for quantifying the linear relationship between two variables. However, its direct application is not suitable for intuitionistic fuzzy numbers due to the presence of membership and non-membership functions. Therefore, in this section, we propose a new parametric correlation coefficient adapted to the intuitionistic fuzzy framework.
Moreover, existing fuzzy extensions of the Pearson correlation coefficient often fail to capture negative relationships and lack flexibility.

$$Cor(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X - E(X))(Y - E(Y))]}{\sigma_X \sigma_Y}. \tag{3.1}$$

In this equation, the symbol $cov$ represents the covariance, $\sigma_X$ represents the standard deviation of the variable $X$, $E(X)$ represents the mean of the variable $X$, and $E$ represents the mathematical expectation. The concept of mathematical expectation refers to the anticipated value of a discrete random variable. It is calculated by summing the product of the probability of each conceivable state occurring and the corresponding value of that state. Pearson's correlation coefficient is a dimensionless index that is stable for the linear transformation of any two variables. In the case of subtraction, the raw data are concentrated by subtracting the mean from each variable, and the sum of the cross products of these concentrated variables is added together. The denominator balances the scales of the variables to have the same units. Therefore, Equation (3.1) shows the correlation coefficient as the concentrated and standardized sum of the cross product of two variables. Using Schwartz's inequality, it is clear that this coefficient is between -1 and 1. If the obtained quantity is positive, it means that the changes of two variables occur in the same direction; that is, with an increase in each variable, the other variable also increases, and on the contrary, if the value of $\rho$ becomes negative, it means that the two variables act in the opposite direction. This indicates that as the value of one variable increases, the value of another variable decreases, and vice versa. If the obtained value becomes zero, it shows that there is no linear relationship between the two variables. A positive correlation of +1 indicates a direct linear relationship, whereas a negative correlation of -1 indicates an inverse linear relationship. Several simple algebraic transformations can be used from this formula for computational tasks.

Usually, the community correlation coefficient $\rho$ and the correlation coefficient of a sample with size $n$ of the population are introduced with $r$. The Pearson correlation coefficient is defined for a statistical sample consisting of $n$ pairs of data $(X_i, Y_i)$ as follows:

$$R = \widehat{Cor(X, Y)} = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}} \tag{3.2}$$

where the average data is defined as $\overline{x} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$. In this section, we first review some of the correlation coefficients that are presented for fuzzy numbers, and then the proposed correlation coefficient. Suppose $C$ and $D$ are two arbitrary IFSs. Hung (2001) introduced the following correlation coefficient:

$$Cor_1(C, D) = \frac{\tau_1(C, D) + \tau_2(C, D)}{2},$$

where

$$\tau_1 = \frac{\sum_{i=1}^{n}(\mu_C(x_i) - \overline{\mu}_C)(\mu_D(x_i) - \overline{\mu}_D)}{\sqrt{\sum_{i=1}^{n}(\mu_C(x_i) - \overline{\mu}_C)^2}\sqrt{\sum_{i=1}^{n}(\mu_D(x_i) - \overline{\mu}_D)^2}}$$

The $\tau_2$ function is written similarly to the $\tau_1$ function for the non-membership function $(\nu)$, and $\overline{\mu}$ and $\overline{\nu}$ denote the membership and non-membership functions, respectively.

Park et al. (2009) proposed another correlation coefficient as follow:

$$Cor_2(C, D) = \frac{\tau_1(C, D) + \tau_2(C, D) + \tau_3(C, D)}{3},$$

that $\tau_1$ and $\tau_2$ functions are similar to Hong's definition, and $\tau_3$ function is similar to $\tau_1$, only with the difference that it is written for hesitant membership function ($\pi$).
Liu et al. (2016) proposed following correlation coefficient:

$$Cor_3(C, D) = \frac{\rho(C, D)}{\sqrt{\phi(C)\phi(D)}},$$

where

$$\phi(X) = \frac{1}{n-1}\sum_{i=1}^{n} d_i^2(X), \quad \rho(X, Y) = \frac{1}{n-1}\sum_{i=1}^{n} d_i(X)d_i(Y),$$
$$d_i(X) = (\mu_X(x_i) - \overline{\mu}_X) - (\nu_X(x_i) - \overline{\nu}_X).$$

Thao et al. (2019) modified the proposed formula by Liu et al. (2016) as follow:

$$Cor_4(C, D) = \frac{\rho(C, D)}{\sqrt{\phi(C)\phi(D)}},$$

where

$$\phi(X) = \frac{1}{n-1}\sum_{i=1}^{n}[(\mu_X(x_i) - \overline{\mu}_X)^2 + (\nu_X(x_i) - \overline{\nu}_X)^2],$$
$$\rho(X, Y) = \frac{1}{n-1}\sum_{i=1}^{n}[(\mu_X(x_i) - \overline{\mu}_X)(\mu_Y(x_i) - \overline{\mu}_Y) + (\nu_X(x_i) - \overline{\nu}_X)(\nu_Y(x_i) - \overline{\nu}_Y)].$$

One important issue in $\alpha$-cut based fuzzy arithmetic is the propagation and accumulation of approximation errors, especially in multi-step computations such as the proposed correlation coefficient. Since the $\alpha$-cut representation converts intuitionistic fuzzy numbers into interval-valued forms, repeated operations (such as subtraction, integration, and normalization) may enlarge the uncertainty bounds and introduce numerical inaccuracies.

### 3.1 Proposed correlation coefficient

Before presenting the proposed correlation coefficient, we define the notation used for intuitionistic fuzzy numbers (IFNs). For an IFN $\tilde{a}_i$, we denote its membership and non-membership functions in terms of their $\alpha$-cuts and $\beta$-cuts as:

$$\tilde{a}_i = (\underline{u}_{ai}, \overline{u}_{ai}, \underline{v}_{ai}, \overline{v}_{ai}),$$

where:

- $\underline{u}_{ai}$ and $\overline{u}_{ai}$ are the lower and upper bounds of the membership function at level $\alpha$,
- $\underline{v}_{ai}$ and $\overline{v}_{ai}$ are the lower and upper bounds of the non-membership function at level $\beta$.

For triangular intuitionistic fuzzy numbers (TriIFNs), we use the parametric representation:

$$\tilde{A} = (m, p, q, p', q'),$$

where:

- $m$ is the modal (central) value,
- $p$ and $q$ are the left and right spreads of the membership function,
- $p'$ and $q'$ are the left and right spreads of the non-membership function,
- with the conditions $0 \leq p' \leq p$ and $0 \leq q' \leq q$ to ensure $\mu(x) + \nu(x) \leq 1$.

Now, consider a fuzzy statistical sample with $n$ pairs of fuzzy data $(\tilde{A}, \tilde{B}) = \{(\tilde{a}_i, \tilde{b}_i) | \tilde{a}_i = (\underline{u}_{ai}, \overline{u}_{ai}, \underline{v}_{ai}, \overline{v}_{ai}),\ \tilde{b}_i = (\underline{u}_{bi}, \overline{u}_{bi}, \underline{v}_{bi}, \overline{v}_{bi}), i = 1, 2, \cdots, n\}$. The parametric correlation coefficient between two classes of the fuzzy data based on (3.2) is defined as follows:

$$Cor(\tilde{A}, \tilde{B})_{\alpha,\beta} = \frac{\sum_{i=1}^{n} d(\tilde{a}_i, \overline{\tilde{a}})\, d(\tilde{b}_i, \overline{\tilde{b}})}{\sqrt{\sum_{i=1}^{n} d(\tilde{a}_i, \overline{\tilde{a}})^2}\ \sqrt{\sum_{i=1}^{n} d(\tilde{b}_i, \overline{\tilde{b}})^2}} \tag{3.3}$$

The sample mean of intuitionistic fuzzy numbers is defined as:

$$\overline{\tilde{a}} = \left(\underline{u}_{\overline{a}}, \overline{u}_{\overline{a}}, \underline{v}_{\overline{a}}, \overline{v}_{\overline{a}}\right),$$

where

$$\underline{u}_{\overline{a}} = \frac{1}{n}\sum_{i=1}^{n} \underline{u}_{ai}, \quad \overline{u}_{\overline{a}} = \frac{1}{n}\sum_{i=1}^{n} \overline{u}_{ai}, \quad \underline{v}_{\overline{a}} = \frac{1}{n}\sum_{i=1}^{n} \underline{v}_{ai}, \quad \overline{v}_{\overline{a}} = \frac{1}{n}\sum_{i=1}^{n} \overline{v}_{ai},$$

, and the fuzzy difference operator $d$ is defined as follows:

$$d(\tilde{a}, \tilde{b}) = \left[\int_{\alpha}^{1} \left(\overline{u}_{\tilde{a}} - \overline{u}_{\tilde{b}}\right) + \left(\underline{u}_{\tilde{a}} - \underline{u}_{\tilde{b}}\right) dr + \int_{0}^{\beta} \left(\underline{v}_{\tilde{a}} - \underline{v}_{\tilde{b}}\right) + \left(\overline{v}_{\tilde{a}} - \overline{v}_{\tilde{b}}\right) dr\right] \tag{3.4}$$

For triangular intuitionistic fuzzy numbers $\tilde{a} = (m_1, p_1, q_1, p_1', q_1')$ and $\tilde{b} = (m_2, p_2, q_2, p_2', q_2')$, their $\alpha$-cut and $\beta$-cut representations based on Equation (2.1) are given by:

$$\underline{u}_{\tilde{a}}(r) = m_1 - p_1(1 - r), \qquad \overline{u}_{\tilde{a}}(r) = m_1 + q_1(1 - r),$$

$$\underline{u}_{\tilde{b}}(r) = m_2 - p_2(1 - r), \qquad \overline{u}_{\tilde{b}}(r) = m_2 + q_2(1 - r),$$

$$\underline{v}_{\tilde{a}}(r) = m_1 - p_1'(1-r), \qquad \overline{v}_{\tilde{a}}(r) = m_1 + q_1'(1-r),$$

$$\underline{v}_{\tilde{b}}(r) = m_2 - p_2'(1-r), \qquad \overline{v}_{\tilde{b}}(r) = m_2 + q_2'(1-r).$$

Substituting these expressions into Equation (3.4), we obtain

$$\begin{aligned} d(\tilde{a},\tilde{b}) = \int_\alpha^1 \Big[&(m_1 - p_1(1-r)) - (m_2 - p_2(1-r)) \\ &+ (m_1 + q_1(1-r)) - (m_2 + q_2(1-r))\Big]dr \\ + \int_0^\beta \Big[&(m_1 - p_1'(1-r)) - (m_2 - p_2'(1-r)) \\ &+ (m_1 + q_1'(1-r)) - (m_2 + q_2'(1-r))\Big]dr. \end{aligned}$$

After simplification, we have:

$$\begin{aligned} d(\tilde{a},\tilde{b}) &= \int_\alpha^1 \Big[2(m_1 - m_2) + (p_2 - p_1 + q_1 - q_2)(1-r)\Big]dr \\ &+ \int_0^\beta \Big[2(m_1 - m_2) + (p_2' - p_1' + q_1' - q_2')(1-r)\Big]dr. \end{aligned}$$

Evaluating the integrals yields:

$$\begin{aligned} d(\tilde{a},\tilde{b}) = &\, 2(m_1 - m_2)(1 - \alpha + \beta) \\ &+ (p_2 - p_1 + q_1 - q_2)\frac{(1-\alpha)^2}{2} \\ &+ (p_2' - p_1' + q_1' - q_2')\frac{1-(1-\beta)^2}{2}, \end{aligned}$$

which gives Equation (3.5).

If $\tilde{a} = (m_1, p_1, q_1, p_1', q_1')$ and $\tilde{b} = (m_2, p_2, q_2, p_2', q_2')$ are two intuitionistic triangular fuzzy numbers (using the parametric representation defined above), the value of $d(\tilde{a},\tilde{b})$ is:

$$\begin{aligned} d(\tilde{a},\tilde{b}) = 2(m_1 - m_2)(1 - \alpha + \beta) + (p_2 - p_1 + q_1 - q_2)\frac{(1-\alpha)^2}{2} \\ + (p_2' - p_1' + q_1' - q_2')\frac{1-(1-\beta)^2}{2} \end{aligned} \tag{3.5}$$

which, as can be seen, is a function of $\alpha$ and $\beta$. By incorporating the parameters $\alpha$ and $\beta$ this measure allows flexible control over the contribution of membership and non-membership components Among the available approaches for fuzzy arithmetic, such as the extension principle and function principle, the $\alpha$-cut method is adopted in this study due to its computational simplicity and practical tractability. While the extension principle provides a theoretically exact framework, it often leads to complex

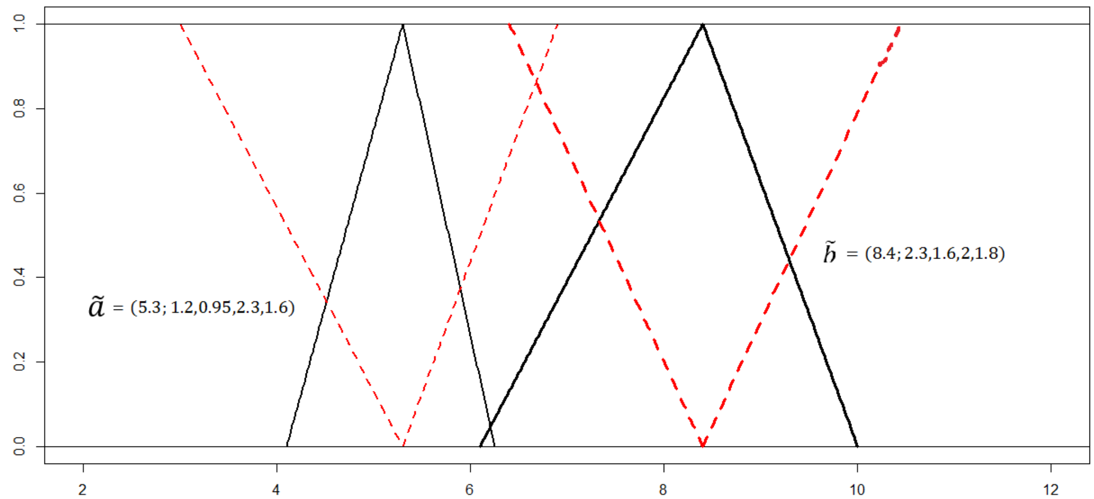


Figure 2: Triangular intuitionistic fuzzy number $\tilde{a} = (5.3, 1.2, 0.95, 2.3, 1.6)$ and $\tilde{b} = (8.4; 2.3, 1.6, 2, 1.8)$.

nonlinear optimization problems, especially in multi-step computations such as correlation analysis. In contrast, the $\alpha$-cut representation transforms fuzzy numbers into interval forms, enabling efficient numerical implementation and analytical derivation. It is important to note that the $\alpha$-cut approach may introduce approximation errors compared to exact methods based on the extension principle. However, this trade-off is justified by the significant reduction in computational complexity and the ability to handle large-scale or parametric problems efficiently. Therefore, the proposed method prioritizes computational feasibility while maintaining acceptable accuracy for practical applications.
However, in the proposed method, the use of integral-based aggregation over $\alpha$ and $\beta$ levels (see Equation (3.4)) plays a stabilizing role. Instead of relying on a single $\alpha$-cut, the integration over intervals reduces the sensitivity to local fluctuations and mitigates the impact of error accumulation.

**Example 3.1.** Let $\tilde{a} = (5.3, 1.2, 0.95, 2.3, 1.6)$ and $\tilde{b} = (8.4; 2.3, 1.6, 2, 1.8)$ are two intuitionistic triangular fuzzy numbers as illustrated in Figure 2, the value of $d(\tilde{a}, \tilde{b})$ for different alpha and beta is recorded in Table 1. Also, the changes of $d(\tilde{a}, \tilde{b})$ compared to alpha and beta are plotted in Figure 3. As you can see, increasing alpha increases the value of $d(\tilde{a}, \tilde{b})$, while increasing the value of beta decreases the value of $d(\tilde{a}, \tilde{b})$.

A sensitivity analysis of the proposed distance measure (Eqs. (3.4), (3.5)) shows that the output is directly influenced by the parameters $\alpha$ and $\beta$. Specifically, the terms involving $(1 - \alpha)$ and $\beta$ in Equation (3.5) indicate that variations in these parameters systematically affect the magnitude of the distance. As illustrated in Table 1 and Figure 3, increasing $\alpha$ generally increases the contribution of the membership component, while increasing $\beta$ reduces the influence of the non-membership component.

**Theorem 3.2.** *For both fuzzy statistical samples* $\tilde{A} = \{\tilde{a}_i = (\underline{u}_{ai}, \overline{u}_{ai}, \underline{v}_{ai}, \overline{v}_{ai}),\ i = 1, 2, \cdots, n\}$

Table 1: The fuzzy difference operator for $\tilde{a}$ and $\tilde{b}$.

| | $\alpha$ | $\beta$ | $d(\tilde{a},\tilde{b})$ |
|---|---|---|---|
| 1 | 0.2 | 0.8 | −10.016 |
| 2 | 0.8 | 0.2 | −2.561 |
| 3 | 0.3 | 0.4 | −6.8697 |
| 4 | 0.4 | 0.3 | −5.6265 |
| 5 | 0.5 | 0.5 | −6.3312 |
| 6 | 0.1 | 0.8 | −10.5977 |
| 7 | 0.1 | 0.9 | −11.2252 |
| 8 | 0.99 | 0.01 | −0.1289 |

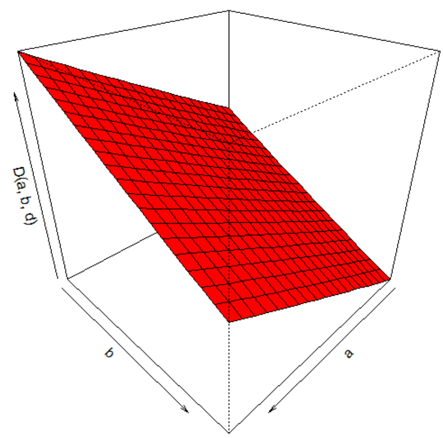


Figure 3: Changes of $d(\tilde{a},\tilde{b})$ for different $\alpha$ and $\beta$ in Example 3.1.

*and* $\tilde{B} = \{\tilde{b}_i = (\underline{u}_{bi}, \overline{u}_{bi}, \underline{v}_{bi}, \overline{v}_{bi}),\ i = 1, 2, \cdots, n\}$ *the following relationships are established for the introduced correlation coefficient:*

*1 :* $Corr(\tilde{A},\tilde{B}) = Corr(\tilde{B},\tilde{A})$

*2 :If* $\tilde{A} = \tilde{B}$ *then* $Corr(\tilde{A},\tilde{B}) = 1$

*3 :* $-1 \leq Corr(\tilde{A},\tilde{B}) \leq +1$

*Proof.* The proof of parts 1 and 2 is clear according to relation (3.3). Here we only prove part 3. Cauchy-Schwarz's inequality for $a_1, a_2, \cdots, a_n$ and $b_1, b_2, \cdots, b_n$ values is defined as follows:

$$(\sum_{i=1}^{n} a_i b_i)^2 \leq \sum_{i=1}^{n}(a_i^2).\sum_{i=1}^{n}(b_i^2)$$

on the other hand

$$Cor(\tilde{A},\tilde{B})^2_{\alpha,\beta} = \frac{(\sum_{i=1}^{n} d(\tilde{a}_i,\bar{\tilde{a}})d(\tilde{b}_i,\bar{\tilde{b}}))^2}{\sum_{i=1}^{n} d(\tilde{a}_i,\bar{\tilde{a}})^2 \sum_{i=1}^{n} d(\tilde{b}_i,\bar{\tilde{b}})^2}$$

And by using Cauchy-Schwarz's inequality, it is clear that the numerator of the above relation is smaller than the denominator of the fraction, or we have:

$$(\sum_{i=1}^{n} d(\tilde{a}_i, \bar{\tilde{a}})d(\tilde{b}_i, \bar{\tilde{b}}))^2 \leq \sum_{i=1}^{n} d(\tilde{a}_i, \bar{\tilde{a}})^2 \sum_{i=1}^{n} d(\tilde{b}_i, \bar{\tilde{b}})^2$$

And as a result we have:

$$Cor(\tilde{A}, \tilde{B})^2_{\alpha,\beta} \leq 1$$

that shows:

$$-1 \leq Cor(\tilde{A}, \tilde{B})_{\alpha,\beta} \leq 1$$

This result follows from the definition of the proposed correlation coefficient, where opposite contributions of membership and non-membership components lead to the minimum value. □

*Remark* 3.3. The correlation coefficient equals −1 when the two intuitionistic fuzzy numbers are completely negatively correlated. This occurs when the membership function of one IFN coincides with the non-membership function of the other, and conversely, indicating an inverse relationship.
Equality $Cor(\tilde{A}, \tilde{B}) = -1$ holds when there exists a negative linear dependence between the corresponding fuzzy difference operators, i.e., $d(\tilde{a}_i, \tilde{a}) = -\lambda d(\tilde{b}_i, \tilde{b})$ for some $\lambda > 0$.

The correlation coefficient defined in this section has two important features. First, unlike previously introduced correlation coefficients which are between zero and one, the correlation coefficient introduced in this article is between -1 and 1, and therefore this correlation coefficient has the ability to show the dependence Measure positive and negative between intuitionistic fuzzy numbers. On the other hand, this correlation coefficient is parameterized and the decision maker can choose the decision level. For example, one can find the correlation coefficient between fuzzy numbers that have a high degree of membership.

## 4 Practical examples

In this section, two practical examples related to the parametric correlation coefficient are presented.

### 4.1 Medical diagnosis

There are three categories in disease diagnosis: symptom set $S = \{S_1, S_2, \cdots, S_n\}$, diagnosis set $D = \{D_1, D_2, \cdots, D_m\}$, and patient set $P = \{P_1, P_2, \cdots, P_l\}$.
Diagnosing diabetes and knowing the high probability of contracting this disease is not an easy task because the symptoms of this disease are similar to some other diseases. Here we are going to solve an example related to this disease. There are two types of diabetes: type 1 diabetes and type 2 diabetes. one of which requires insulin injections,

Table 2: Characterization of symptoms for diseases.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ |
|---|---|---|---|---|---|
| Type 1 diabetes | $(6.8; 1.7, 1.4, 1.2, 2.3)$ | $(8.3; 0.8, 0.8, 1.9, 2.5)$ | $(8.6; 1.4, 0.3, 2.3, 1.6)$ | $(8.4; 0.8, 0.7, 1.6, 1.4)$ | $(7.6; 2.3, 1, 2, 1.8)$ |
| Type 2 diabetes | $(7.2; 1.1, 1.4, 1.2, 2.3)$ | $(8.6; 0.6, 0.6, 1.9, 2.5)$ | $(8.2; 0.6, 0.5, 2.3, 1.6)$ | $(9; 0.4, 0.8, 1.6, 1.4)$ | $(5.8; 1.2, 1, 2, 1.8)$ |

Table 3: Characterization of symptoms for patients.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ |
|---|---|---|---|---|---|
| Amelia | $(8.1, 8.6, 9.6)$ | $(7.9, 8.6, 9.3)$ | $(8.1, 9.2, 9.6)$ | $(8, 8.9, 9.6)$ | $(8.2, 8.5, 9.2)$ |
| Emily | $(8.3, 8.7, 9.3)$ | $(8.6, 8.9, 9.2)$ | $(8, 8.6, 9.6)$ | $(8.4, 8.9, 9.6)$ | $(7.1, 8.1, 9.2)$ |
| Sara | $(7.6, 8.3, 8.7)$ | $(8.6, 8.7, 8.8)$ | $(7.8, 8.3, 8.9)$ | $(8.4, 8.6, 9)$ | $(4.8, 5.6, 6.5)$ |
| Ryan | $(6.8, 7.6, 8.6)$ | $(7.8, 8.6, 9.4)$ | $(8.3, 9, 9.5)$ | $(8.3, 9, 9.5)$ | $(5, 6, 7)$ |

but the other can be controlled with medication. The three categories of data mentioned above for this disease are defined as follows:

$$
\begin{aligned}
&S = \{S_1,\ S_2\} = \{Type\,1\,diabetes,\ Type\,2\,diabetes\},\\
&D = \{D_1,\ D_2,\ D_3,\ D_4,\ D_5\}\\
&\quad = \{Weight\,loss,\ Blurred\,vision,\ Dry\,mouth,\ Frequent\,urination,\ Extreme\,fatigue\}\\
&P = \{P_1,\ P_2,\ P_3,\ P_4\} = \{Amelia,\ Emily,\ Sara,\ Ryan\}
\end{aligned}
$$

The mean of type 1 diabetes is $(7.94; 1.4,\ 0.84, 1.8, 1.92)$ and for type 2 is $(7.76; 0.78, 0.86, 1.8, 1.92)$. The values presented in Table 3 can be interpreted as a special case of the parametric representation of intuitionistic fuzzy numbers. Specifically, each triangular value is considered in the form $(m, p, q, p', q')$, where the spreads are assumed to be symmetric, i.e., $p = q$ and $p' = q'$. This assumption simplifies the representation while remaining consistent with the general framework introduced in Definition 2.6.
The membership functions characterizing the symptoms for type 1 and type 2 diabetes are shown in Figure 4. This transformation allows us to apply the proposed correlation coefficient without loss of generality. Table 4 shows the correlation coefficient between two types of diseases and symptoms of 4 patients Amelia, Emily Sara and Ryan. As you can see, there is a very high correlation (0.96) between the symptoms of Ryan's disease and type 2 diabetes. After Ryan, Emily and Sara also have a high correlation coefficient with type 2 diabetes symptoms. As can be seen, Ryan has a significant correlation with the symptoms of type 1 diabetes. By comparing the correlation coefficients between the symptoms of two types of diabetes (Table 5), we find that there is a direct and relatively high correlation between the symptoms of these two types of diseases (0.7), which can partially justify the cause of this. On the other hand, by comparing the correlation coefficient between the disease symptoms in different patients (4 patients), as shown in Table 7 that there is a high correlation between the disease symptoms of Ryan, Emily and Sara. which confirms the results of Table 4.

Table 4: Correlation coefficient between two types of diabetess type1 and type2 and symptoms of 4 patients given in Table 3 .

| Type1 | $\alpha = 0.99, \beta = 0.01$ | $\alpha = 0, \beta = 0.8$ | $\alpha = 0.3, \beta = 0.4$ |
|---|---|---|---|
| Amelia | 0.7060 | 0.5920 | 0.6192 |
| Emily | 0.2607 | 0.3910 | 0.3680 |
| Sara | 0.2658 | 0.3853 | 0.3638 |
| Ryan | 0.6303 | 0.6878 | 0.6779 |
| Type 2 | $\alpha = 0.99, \beta = 0.01$ | $\alpha = 0, \beta = 0.8$ | $\alpha = 0.3, \beta = 0.4$ |
| Amelia | 0.6097 | 0.5306 | 0.5472 |
| Emily | 0.8877 | 0.9301 | 0.9230 |
| Sara | 0.8728 | 0.9079 | 0.9020 |
| Ryan | 0.9672 | 0.9624 | 0.9638 |

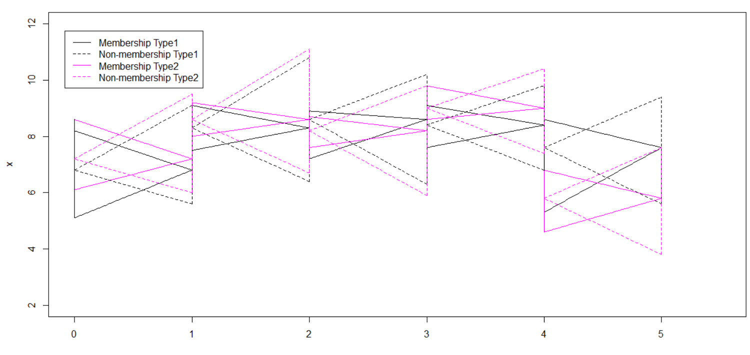


Figure 4: Membership functions characterizing the symptoms for diseases type1 and type2.

## 4.2 MCDM

In this section, the candidate selection problem is presented to show the ability of the parametric correlation coefficient. The following criteria have been chosen to choose the best alternative: $C_1$ : Language communication skills, $C_2$ : Work history and personal experiences, $C_3$ : Self Confidence, $C_4$ : marketing strategy. To choose the best option, we first select the ideal alternative (the option that has the best value in each criterion). Because in order to identify the ideal alternative, we need to compare IFNs, so to do this, we use the method suggested in Darehmiraki (2019). The values presented in Table 8 are expressed in a simplified triangular form. These can be interpreted as a special case of intuitionistic fuzzy numbers in the parametric form $(m, p, q, p', q')$, where symmetric spreads are assumed, i.e., $p = q$ and $p' = q'$. This transformation enables the direct application of the proposed correlation coefficient within the intuitionistic fuzzy framework. The ranking criterion for triangular intuitionistic fuzzy numbers is used as follows.

$$r(\tilde{a}) = \frac{\left[\int_0^1 (\overline{u}_{\tilde{a}} + \underline{u}_{\tilde{a}})dr + \int_0^1 (\underline{v}_{\tilde{a}} + \overline{v}_{\tilde{a}})dr\right]}{2} \tag{4.1}$$

Table 5: Correlation coefficient between symptoms of type 1 and type 2 diabetes.

| | $\alpha$ | $\beta$ | $Cor(T1, T2)$ |
|---|---|---|---|
| 1 | 0.2 | 0.8 | 0.7038 |
| 2 | 0.8 | 0.2 | 0.6789 |
| 3 | 0.3 | 0.4 | 0.7083 |
| 4 | 0.4 | 0.3 | 0.7028 |
| 5 | 0.5 | 0.5 | 0.6916 |
| 6 | 0.1 | 0.8 | 0.7111 |
| 7 | 0.1 | 0.9 | 0.7078 |
| 8 | 0.99 | 0.01 | 0.6706 |

Table 6: Correlation coefficient between disease symptoms in different patients for 0.99 $\beta = 0.01$.

| | Amelia | Emily | Sara | Ryan |
|---|---|---|---|---|
| Amelia | 1 | 0.3380 | 0.4359 | 0.6383 |
| Emily | 0.3380 | 1 | 0.9822 | 0.9017 |
| Sara | 0.4359 | 0.9822 | 1 | 0.9314 |
| Ryan | 0.6383 | 0.9017 | 0.9314 | 1 |

Table 7: Correlation coefficient between disease symptoms in different patients for $\alpha = 0.3$ and $\beta = 0.8$.

| | Amelia | Emily | Sara | Ryan |
|---|---|---|---|---|
| Amelia | 1 | 0.3388 | 0.4336 | 0.6327 |
| Emily | 0.3388 | 1 | 0.9827 | 0.9031 |
| Sara | 0.4336 | 0.9827 | 1 | 0.9321 |
| Ryan | 0.6327 | 0.9031 | 0.9321 | 1 |

Table 8: Characterization of criteria for alternatives.(Values expressed in triangular form (interpreted as symmetric intuitionistic fuzzy numbers)).

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | $(7.1, 8.2, 9)$ | $(8.1, 8.9, 9.5)$ | $(8.5, 9, 9.6)$ | $(8.3, 9.2, 9.6)$ |
| $A_2$ | $(6.2, 7.8, 8.9)$ | $(5.2, 6.2, 7)$ | $(4.2, 5.3, 6.7)$ | $(4.1, 5.1, 6.3)$ |
| $A_3$ | $(6.1, 7.2, 8)$ | $(7.6, 8.5, 8.9)$ | $(8.1, 8.9, 9.5)$ | $(7.5, 8.5, 9.6)$ |
| $A_4$ | $(7.5, 8.1, 9.3)$ | $(8, 8.7, 9.6)$ | $(8.5, 8.9, 9.7)$ | $(8.1, 8.6, 9.4)$ |
| $A_5$ | $(5.2, 7.1, 8.2)$ | $(5.2, 6.5, 8.2)$ | $(4.8, 6.5, 7.8)$ | $(5.2, 7.1, 8.5)$ |

which for triangular intuitionistic fuzzy number $\tilde{A} = (m, p, q, p', q')$ is equal to:

$$r(\tilde{a}) = \frac{4m + \dfrac{q + q' - (p + p')}{2}}{2} \tag{4.2}$$

In Table 9, with the help of formula (4.2), the ideal alternative can be found through IFNs ranking. In Table 10, the correlation coefficient between other alternatives and

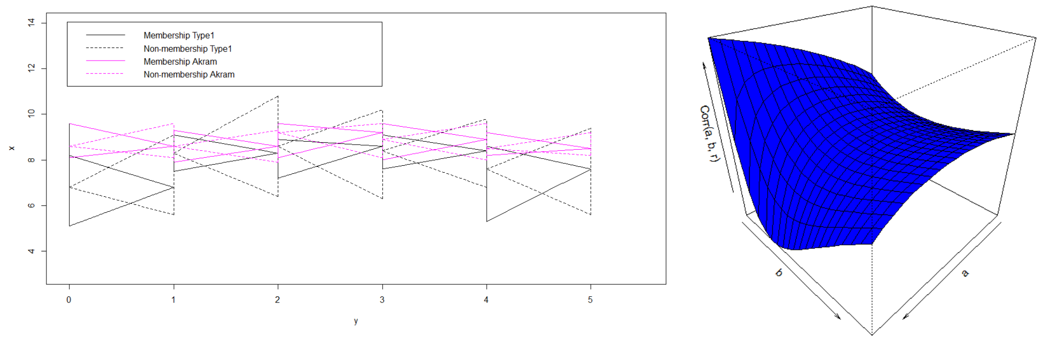


Figure 5: Membership functions of Amelia and diabetes Type1 (left) and correlation coefficient between them for different alpha and beta (right)

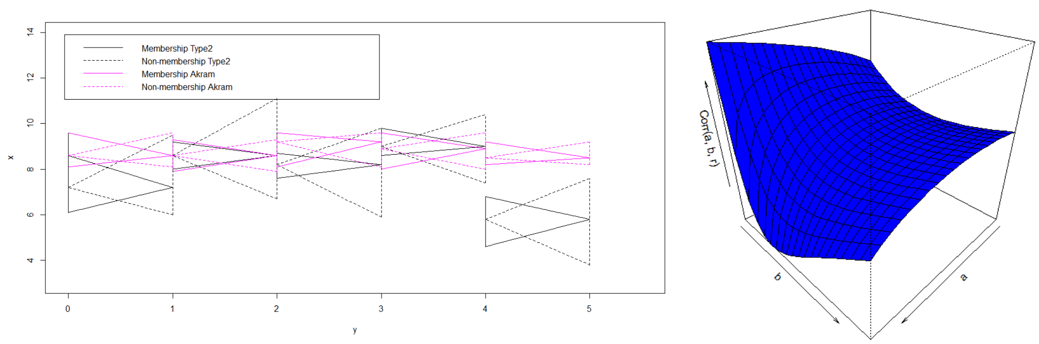


Figure 6: Membership functions of Amelia and diabetes Type2 (left) and correlation coefficient between them for different alpha and beta (right)

Table 9: Ranking the criteria to achieve the ideal alternative.

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $A_1$ | 16.25 | 17.7 | 18.05 | 18.15 |
| $A_2$ | 15.35 | 12.3 | 10.75 | 10.3 |
| $A_3$ | 14.25 | 16.75 | 17.7 | 17.05 |
| $A_4$ | 16.5 | 17.5 | 18 | 17.35 |
| $A_5$ | 13.8 | 13.2 | 12.8 | 13.95 |
| $A^\star$ | $(7.5, 8.1, 9.3)$ | $(8.1, 8.9, 9.5)$ | $(8.5, 9, 9.6)$ | $(8.3, 9.2, 9.6)$ |

the ideal alternative is calculated for different values of $\alpha$ and $\beta$. As you can see, in this example, when the values of $\alpha$ and $\beta$ change, the correlation coefficients do not change much. The first alternative has the highest correlation coefficient with the ideal, and the second and fifth options have a negative correlation coefficient. In Table 11, the correlation coefficient between the options is calculated. In Figure 9, a comparison has been made between the membership functions of the first alternative and the ideal alternative, and the correlation coefficients between them have been calculated for different values of $\alpha$ and $\beta$. In Figure 10, the same has been done for the second alternative

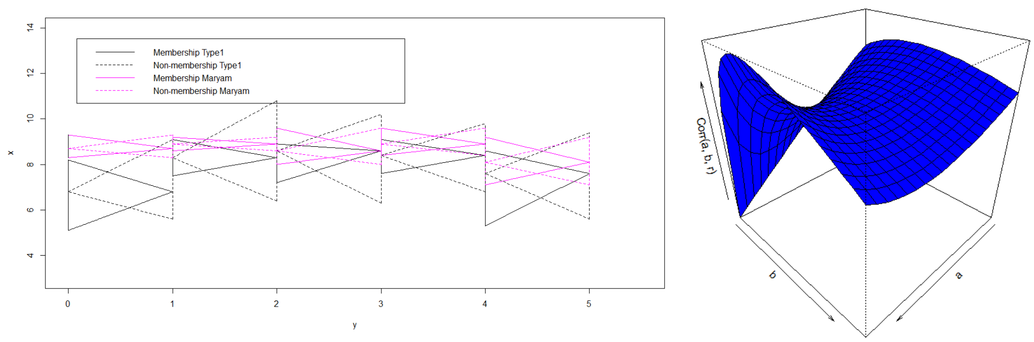


Figure 7: Membership functions of Emily and diabetes Type1 (left) and correlation coefficient between them for different alpha and beta (right).

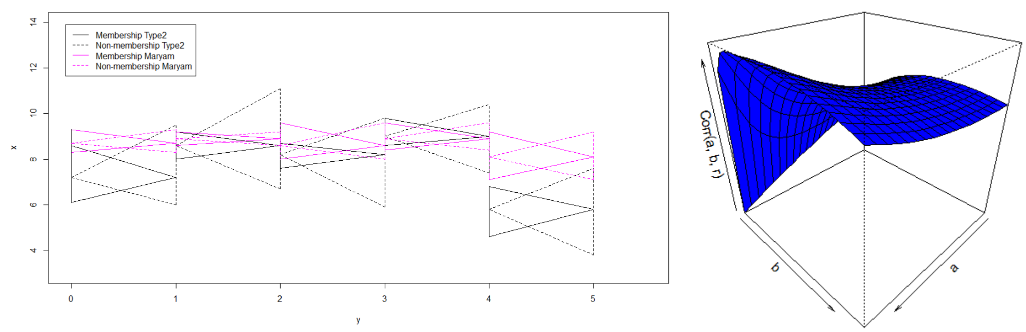


Figure 8: Membership functions of Emily and diabetes Type2 (left) and correlation coefficient between them for different alpha and beta (right).

Table 10: Correlation coefficient between characterization of other alternatives and Ideal alternative.

| | $\alpha = 0.99, \beta = 0.01$ | $\alpha = 0.1, \beta = 0.8$ | $\alpha = 0.3, \beta = 0.4$ |
|---|---|---|---|
| $A_1$ | 0.9993 | 0.9992 | 0.9993 |
| $A_2$ | −0.9919 | −0.9921 | −0.9921 |
| $A_3$ | 0.9722 | 0.9740 | 0.9734 |
| $A_4$ | 0.8758 | 0.8772 | 0.8767 |
| $A_5$ | −0.3516 | −0.3518 | −0.3517 |

and the ideal alternative. Figure 11 shows the membership functions of A5 and the ideal alternative. Furthermore, the numerical results presented in Tables 4–7 indicate that variations in $\alpha$ and $\beta$ do not significantly affect the final correlation values. This suggests that the proposed coefficient is relatively robust against small perturbations in $\alpha$-cut levels, and therefore the accumulation of computational errors is practically limited.

Despite these variations at the distance level, the resulting correlation coefficients remain relatively stable across different values of $\alpha$ and $\beta$ (see Tables 4–7). This indicates

Table 11: Correlation coefficient between different alternatives in Table 8 for $\alpha = 0.99$ $\beta = 0.01$.

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| $A_1$ | 1 | $-0.9867$ | 0.9751 | 0.8820 | $-0.3673$ |
| $A_2$ | $-0.9867$ | 1 | $-0.9518$ | $-0.8441$ | 0.2919 |
| $A_3$ | 0.9751 | $-0.9518$ | 1 | 0.9644 | $-0.5605$ |
| $A_4$ | 0.8820 | $-0.8441$ | 0.9644 | 1 | $-0.7581$ |
| $A_5$ | $-0.3673$ | 0.2919 | $-0.5605$ | $-0.7581$ | 1 |

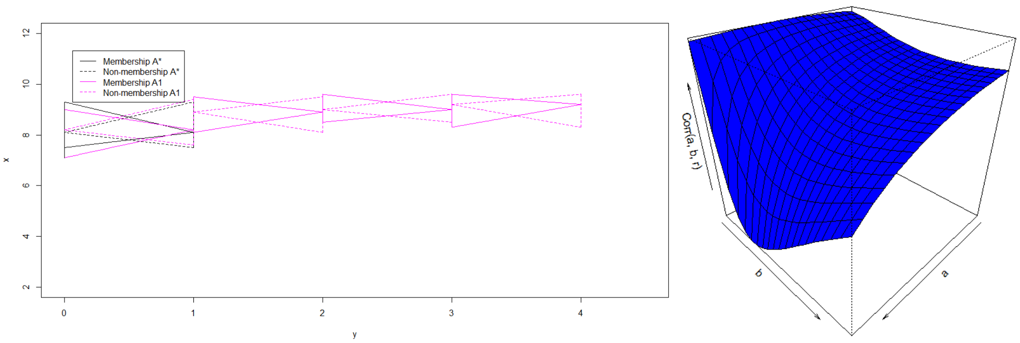


Figure 9: Comparison of membership function of different criteria for the alternative $A_1$ and the ideal alternative (left) and correlation coefficient between them for different alpha and beta (right).

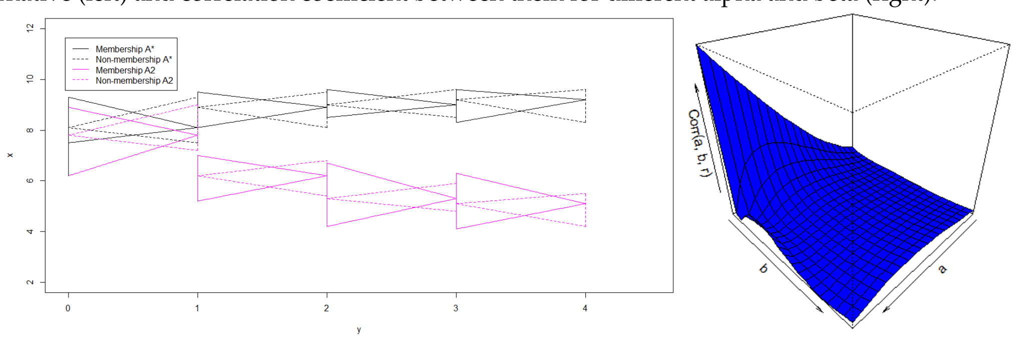


Figure 10: Comparison of membership function of different criteria for the alternative $A_2$ and the ideal alternative (left) and correlation coefficient between them for different alpha and beta (right).

that although local changes in $\alpha$-cut levels affect intermediate computations, their overall impact on the final correlation measure is limited.
Therefore, the proposed correlation coefficient demonstrates robustness with respect to parameter variations, and the propagation of computational errors does not significantly distort the final results.

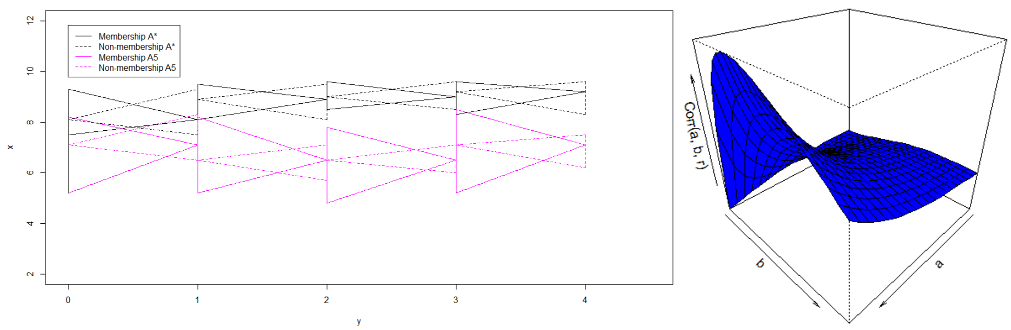


Figure 11: Membership functions of A5 and Ideal state (left) and correlation coefficient between them for different alpha and beta (right).

Table 12: Comparison of existing similarity measures with the proposed method

| Method | Range | Negative Corr | Parametric | Based on | Flexibility |
|---|---|---|---|---|---|
| Hung (2001) | [0,1] | No | No | $\mu, \nu$ | Low |
| Park et al. (2009) | [0,1] | No | No | $\mu, \nu, \pi$ | Medium |
| Liu and Kao (2002) | [0,1] | No | No | distance | Medium |
| Thao (2020) | [0,1] | No | No | $\mu, \nu$ | Medium |
| **Proposed** | [-1,1] | **Yes** | **Yes** | distance + $\alpha, \beta$ | **High** |

# 5 Comprehensive Comparative Analysis

A key limitation of many existing correlation coefficients (e.g., Hung (2001), Park et al. (2009), Liu and Kao (2002), and Thao (2020)) is that their values are restricted to the interval $[0, 1]$, which prevents them from capturing negative relationships. In contrast, the proposed method produces values in the range $[-1, 1]$, enabling the detection of both positive and negative dependencies.

Unlike existing approaches that rely primarily on direct aggregation of membership and non-membership functions, the proposed method is based on a parametric distance operator. This allows the decision maker to adjust the level of analysis through $\alpha$ and $\beta$ parameters, providing additional flexibility that is not available in conventional methods. The numerical results further confirm that the proposed method produces stable and consistent correlation values across different parameter settings, while preserving interpretability in practical applications such as medical diagnosis and decision-making.

Overall, the proposed correlation coefficient offers several advantages over existing methods, including the ability to represent negative correlations, parametric flexibility, and robustness with respect to parameter variations.

## 6 Conclusion

Analyzing the relationships among variables through correlation and regression analysis plays an important role in data analysis and decision-making processes. However, when the available information is imprecise, uncertain, or fuzzy, classical statistical tools may not adequately describe the underlying relationships.
In this paper, a novel parametric distance measure between intuitionistic fuzzy numbers (IFNs) was introduced and subsequently used to extend the Pearson correlation coefficient to the intuitionistic fuzzy environment. One important advantage of the proposed correlation coefficient is its flexibility with respect to decision levels, since the parameters $\alpha$ and $\beta$ allow the decision maker to evaluate correlations under different levels of uncertainty and confidence.
To demonstrate the applicability and effectiveness of the proposed method, several practical examples and applications in multi-criteria decision making and medical diagnosis were presented.
The proposed correlation coefficient possesses the following important features:

1. Unlike many existing fuzzy correlation coefficients, the proposed measure is capable of representing both positive and negative correlations between intuitionistic fuzzy data.

2. The proposed approach provides a parametric framework in which the correlation coefficient can be evaluated at different decision levels by selecting suitable values of $\alpha$ and $\beta$. Consequently, the behavior of the correlation measure can be analyzed under varying degrees of uncertainty.

Future research may include a rigorous sensitivity and error-bound analysis of the proposed $\alpha$-cut based correlation measure, as well as comparisons with alternative representations such as direct membership-function-based and other parametric approaches.

## References

Atanassov KT. Intuitionistic fuzzy sets. Fuzzy Sets and Systems. 1986;20(1):87–96.

Chaudhuri B, Bhattacharya A. On correlation between two fuzzy sets. Fuzzy sets and systems. 2001;118(3):447–456.

Darehmiraki M. A novel parametric ranking method for intuitionistic fuzzy numbers. Iranian Journal of Fuzzy Systems. 2019;16(1):129–143.

Deschrijver G. Arithmetic operators in interval-valued fuzzy set theory. Information sciences. 2007;177(14):2906–2924.

Ejegwa PA, Onyeke IC. Intuitionistic fuzzy statistical correlation algorithm with applications to multicriteria-based decision-making processes. International Journal of Intelligent Systems. 2021;36(3):1386–1407.

Hong DH. Fuzzy measures for a correlation coefficient of fuzzy numbers under TW (the weakest t-norm)-based fuzzy arithmetic operations. Information Sciences. 2006;176(2):150–160.

Hung WL. Using statistical viewpoint in developing correlation of intuitionistic fuzzy sets. International journal of uncertainty, fuzziness and knowledge-based systems. 2001;9(04):509–516.

Kumar M. Intuitionistic fuzzy measures of correlation coefficient of intuitionistic fuzzy numbers under weakest triangular norm. International Journal of Fuzzy System Applications (IJFSA). 2019;8(1):48–64.

Liu B, Shen Y, Mu L, Chen X, Chen L. A new correlation measure of the intuitionistic fuzzy sets. Journal of intelligent & fuzzy systems. 2016;30(2):1019–1028.

Liu ST, Kao C. Fuzzy measures for correlation coefficient of fuzzy numbers. Fuzzy Sets and Systems. 2002;128(2):267–275.

Park JH, Lim KM, Park JS, Kwun YC. Correlation coefficient between intuitionistic fuzzy sets. In: Fuzzy Information and Engineering, Volume 2 Berlin: Springer; 2009. p. 601–610.

Saneifard R, Saneifard R, et al. Correlation coefficient between fuzzy numbers based on central interval. Journal of Fuzzy Set Valued Analysis. 2012;2012:1–9.

Shakouri B, Abbasi Shureshjani R, Daneshian B, Hosseinzadeh Lotfi F. A parametric method for ranking intuitionistic fuzzy numbers and its application to solve intuitionistic fuzzy network data envelopment analysis models. Complexity. 2020;2020(1):6408613.

Taheri S, Zarei R. Extension principle for vague sets and its applications. Advances in Fuzzy Mathematics. 2011;6(1):17–26.

Thao NX. A new correlation coefficient of the Pythagorean fuzzy sets and its applications. Soft Computing. 2020;24(13):9467–9478.

Thao NX, Ali M, Smarandache F. An intuitionistic fuzzy clustering algorithm based on a new correlation coefficient with application in medical diagnosis. Journal of intelligent & fuzzy systems. 2019;36(1):189–198.

Watanabe N, Imaizumi T. A fuzzy correlation coefficient for fuzzy random variables. In: FUZZ-IEEE'99. 1999 IEEE International Fuzzy Systems. Conference Proceedings (Cat. No. 99CH36315), vol. 2 IEEE; 1999. p. 1035–1038.

Ye J. Fuzzy decision-making method based on the weighted correlation coefficient under intuitionistic fuzzy environment. European Journal of Operational Research. 2010;205(1):202–204.

Zadeh LA. Fuzzy sets. Information and control. 1965;8(3):338–353.