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Estimation of Subpopulation Parameters in One-stage Cluster Sampling Design

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Abstract. Sometimes in order to estimate population parameters such as mean and total values, we extract a random sample by cluster sampling method, and after completing sampling, we are interested in using the same sample to estimate the desired parameters in a subset of the population, which is said subpopulation. In this paper, we try to estimate subpopulation parameters in different cases when one-stage cluster sampling design is used.

Keywords. Finite Population, Subpopulation, Cluster Sampling, Unbiased

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1 Introduction

The purpose of a sample survey design is to estimate the population parameters with the lowest cost and the highest accuracy. One of the most used and low cost methods in sampling schemes is cluster sampling, which does not require a population framework

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that it is generally costly. On the other hand, many sampling schemes can be cited where in addition to the inference about the population parameters, it is important to infer the parameters of a part of the population that share a particular feature (subpopulation). Since subpopulation estimators are based on the same sample, there is no additional cost to the researcher. In this paper, we attempt to discuss a subpopulation parameter estimation in a cluster sampling scheme.

To estimate the subpopulation parameters in a simple random sampling scheme, we recall some notion. Suppose that the members of a finite population are $U_1, U_2, \dots, U_N$ and that Y_i denotes the Y-value associated with U_i for $i = 1, \dots, N$. Hence $Y = \sum_{i=1}^{N} Y_i$ and $\overline{Y} = \dfrac{Y}{N}$ represent the total and mean values of the population, respectively. Now, suppose that $u_1, u_2, \dots, u_n$ are sample members and the random variable y_i is the value of the desired attribute u_i for $i = 1, \dots, n$. Consider the attribute c that some members of the population possess, and call them a subpopulation. After simple random sampling for estimating population parameters, we are interested in estimating subpopulation parameters, using the same simple random sample. For this purpose, we assume that N_{c_0} is the size of the subpopulation and that Y_{ci}, for $i = 1, \dots, N_{c_0}$, is the value of the attribute of ith subpopulation member. Then, $Y_c = \sum_{i=1}^{N_{c_0}} Y_{ci}$ and $\overline{Y}_c = \dfrac{Y_c}{N_{c_0}}$ will represent the total and mean values of the subpopulation, respectively. For $j = 1, \dots, n_c$, suppose that the random variable y_{cj} represents the value of the attribute desired for jth sample member in the subpopulation c, where n_c is the sample size in the subpopulation. Obviously n_c is a random variable with hyper-geometric distribution and expectation $\dfrac{nN_{c_0}}{N}$.

To estimate $\overline{Y}_c$, Cochran (1977) introduced the following estimator:

$$\bar{y}_c = \frac{1}{n_c} \sum_{j=1}^{n_c} y_{cj},$$

and

$$V(\bar{y}_c) = S_c^2 [E(\frac{1}{n_c}) - \frac{1}{N_{c_0}}], \tag{1.1}$$

where S_c^2 is the subpopulation variance. Similarly Y_c is estimated by

$$\widehat{Y}_c = N_{c_0} \bar{y}_c, \tag{1.2}$$

but $\widehat{Y}_c$ is an estimator of Y_c if N_{c_0} is known. If N_{c_0} is unknown, by using $\widehat{N}_c = \dfrac{Nn_c}{n}$ as an unbiased estimator for N_{c_0}, the following estimator can be introduced for estimating Y_c:

$$\widehat{\widehat{Y}}_c = \frac{Nn_c}{n}\bar{y}_c. \tag{1.3}$$

We can prove that all estimators $\bar{y}_c$, $\widehat{Y}_c$, and $\widehat{\widehat{Y}}_c$ are unbiased for their corresponding parameters. Let us assume that

$$Y^*_{ci} = \begin{cases} Y_i, & U_i \in c, \\ 0, & o.w. \end{cases} \quad \text{for } i = 1, \ldots, N,$$

and that, in this case,

$$S_c^{*2} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y^*_{ci} - \overline{Y}^*_c\right)^2, \tag{1.4}$$

where $\overline{Y}^*_c$ is the average of Y^*_{ci}'s. Then, it can be shown that

$$V(\widehat{\widehat{Y}}_c) = N^2\frac{S_c^{*2}}{n}(1 - \frac{n}{N}). \tag{1.5}$$

Also, unbiased estimators for $V(\widehat{Y}_c)$ and $V(\widehat{\widehat{Y}}_c)$ are, respectively,

$$v(\widehat{Y}_c) = N_{c_0}^2 s_c^2\left(E\left(\frac{1}{n_c}\right) - \frac{1}{N_{c_0}}\right),$$

and

$$v(\widehat{\widehat{Y}}_c) = N^2\frac{s_c^{*2}}{n}\left(1 - \frac{n}{N}\right),$$

where s_c^2 and s_c^{*2} are the sample variances of y_{ci} and

$$y^*_{ci} = \begin{cases} y_i, & u_i \in c, \\ 0, & o.w. \end{cases}, \tag{1.6}$$

for $i = 1, \ldots, n$.

Note 1. Note that s_c^2 and s_c^{*2} are unbiased estimators for S_c^2 and S_c^{*2}, respectively.

We refer readers to Cochran (1977) and Thompson (2012) for more information on these estimators. Besides, Salehi and Chang (2005) provided another estimator for the parameter of the total subpopulation based on the inverse sampling method and Clark (2009) introduced a regression estimator for total value of subpopulation in two stage sampling. It should be noted that the method of two-stage sampling is difference from that two-stage cluster sampling. Two-stage sampling is useful when the variable of interest y is relatively expensive to measure but a correlated variable x can be measured fairly easily and used to improve the precision of the estimation of parameters of interest. It may also be used to adjust for nonresponse, to sample rare populations, or to improve the sampling frame (see Lohr (2010)). But in two-stage cluster, we cannot census all of elements of clusters selected in the first stage, or it may be expensive to measure all of them. So we select a sample with two-stage cluster sampling.

2 Estimation of Subpopulation Parameters in One-stage Cluster Sampling Design

Consider a population with N clusters that members of the ith cluster with the size of M_i are denoted by $U_{i1}, \ldots, U_{iM_i}$ for $i = 1, \ldots, N$. Assume that $M = \sum_{i=1}^{N} M_i$ is the size of population and that Y_{ij} is the value of the desired attribute for the unit U_{ij}.

The parameters $\tau = \sum_{i=1}^{N} \sum_{j=1}^{M_i} Y_{ij}$ and $\mu = \dfrac{\tau}{M}$ represent the total and the mean of the population, respectively. Accordingly, for $i = 1, \ldots, N$, the parameters $Y_i = \sum_{j=1}^{M_i} Y_{ij}$ and $\overline{Y}_i = \dfrac{Y_i}{M_i}$ represent the total value and the mean of the i-th cluster , respectively. Now for this population, suppose that n clusters are selected under the simple random sampling design and that the clusters are censused. Assume that m_i is the sample size of i-th cluster and that y_{ij} indicates the sample values of the desired attribute in the i-th cluster of sample ($i = 1, \ldots, n$; $j = 1, \ldots, m_i$).

We know that the unbiased estimator of the population total reads as follows:

$$\widehat{\tau} = \frac{N}{n} \sum_{i=1}^{n} y_i = N\overline{y}, \tag{2.1}$$

where $y_i = \sum_{j=1}^{M_i} y_{ij}$, for $i = 1, \ldots, n$, is the total value of the cluster i in the sample.

The variance of this estimator is

$$V(\widehat{\tau}) = N^2 S_b^2 [\frac{1}{n} - \frac{1}{N}], \tag{2.2}$$

where considering $\overline{Y} = \frac{1}{N} \sum_{i=1}^{N} Y_i$, we have

$$S_b^2 = \frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \overline{Y})^2. \tag{2.3}$$

If $s_b^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \overline{y})^2$ is the sample variance $y_1, \ldots, y_n$, then the unbiased estimator $V(\widehat{\tau})$ is

$$v(\widehat{\tau}) = s_b^2 [\frac{1}{n} - \frac{1}{N}]. \tag{2.4}$$

However, the estimator $\widehat{\mu} = \frac{\widehat{\tau}}{M}$ is an unbiased estimator for μ with variance $\frac{V(\widehat{\tau})}{M^2}$.

We also know that $\widehat{\mu}_R = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i}$ is a biased estimator of μ and it is used when M is unknown. The approximate variance of this estimator is

$$V(\widehat{\mu}_R) \simeq \frac{(N-n)}{nN\overline{M}^2(N-1)} \sum_{i=1}^{N} (Y_i - M_i\mu)^2,$$

where $\overline{M} = \frac{1}{N} \sum_{i=1}^{N} M_i$ is the average of the cluster sizes. For more information on ratio estimators, see Rao (1988) and Royal (1988). Moreover, Salehi and Seber (2001) presented an unbiased estimator in adaptive cluster sampling.

Now suppose among the N clusters, N_c clusters have members of the subpopulation. Obviously N_c is a natural number and $1 \leq N_c \leq N$. For $i = 1, \ldots, N_c$, let $M_{c,i}$ be the size of members of the subpopulation in i-th cluster and let $M_c = \sum_{i=1}^{N_c} M_{c,i}$ be the total number of subpopulation members. In concordance with the parameters of the population, let $\tau_c = \sum_{i=1}^{N_c} \sum_{j=1}^{M_{c,i}} Y_{c,ij}$ and $\mu_c = \frac{\tau_c}{M_c}$ represent the total and the mean of the subpopulation, respectively, where $Y_{c,ij}$ is the attribute value for the j-th member of subpopulation in the i-th cluster.

Also, let $Y_{c,i} = \sum_{j=1}^{M_{c,i}} Y_{c,ij}$ and $\overline{Y}_{c,i} = \frac{Y_{c,i}}{M_{c,i}}$ indicate the total value and mean of the subpopulation in i-th cluster for $(i = 1, \dots, N_c)$, respectively, and $\overline{Y}_c = \frac{1}{N_c} \sum_{i=1}^{N_c} Y_{c,i}$ is the average of $Y_{c,i}$'s. Now, if we consider among n clusters in the sample, n_c clusters contain subpopulation's members, then n_c is a random variable with hypergeometric distribution and $E(n_c) = \frac{nN_c}{N}$. Similarly, suppose that $m_{c,i}$ is the sample size of subpopulation's members in the cluster i and that $y_{c,ij}$ represents the sample values of the attribute desired in the i-th cluster containing subpopulation members in the sample $(i = 1, \dots, n_c\,;\ j = 1, \dots, m_{c,i})$. The following theorem presented an unbiased estimator of τ_c.

Theorem 2.1. *a) By considering $y_{c,i} = \sum_{j=1}^{m_i} y_{c,ij}$ as the total value of the subpopulation in the i-th cluster of sample, the estimator*

$$\widehat{\tau}_c = \frac{N_c}{n_c} \sum_{i=1}^{n_c} y_{c,i} = N_c \overline{y}_c,$$

is an unbiased estimator for τ_c.

b) The variance of this estimator is as follows:

$$V(\widehat{\tau}_c) = N_c^2 S_{cb}^2 [E(\frac{1}{n_c}) - \frac{1}{N_c}],$$

where

$$S_{cb}^2 = \frac{1}{N_c - 1} \sum_{i=1}^{N_c} (Y_{ci} - \overline{Y}_c)^2.$$

c) The unbiased estimator for the variance in part b) is

$$v(\widehat{\tau}_c) = s_{cb}^2 [E(\frac{1}{n_c}) - \frac{1}{N_c}],$$

where

$$s_{cb}^2 = \frac{1}{n_c - 1} \sum_{i=1}^{n_c} (y_{c,i} - \overline{y}_c)^2.$$

Proof. a) We know that by given nc, every possible combination of nc of the Nc subpopulation units has equal probability of being included in the sample, so

$$\begin{aligned} E(\widehat{\tau}_c) &= E(\frac{N_c}{n_c}\sum_{j=1}^{n_c} y_{c,i}) \\ &= N_c E[E(\frac{1}{n_c}\sum_{j=1}^{n_c} y_{c,i}|n_c)] \\ &= N_c E(\overline{Y}_c) \\ &= \tau_c. \end{aligned}$$

b) Using the well-known decomposition of $Var(X) = E(Var(X|n_c)) + Var(E(X|n_c))$, we can write

$$\begin{aligned} V(\widehat{\tau}_c) &= N_c^2[E(V(\frac{1}{n_c}\sum_{j=1}^{n_c} y_{c,i}|n_c)) + V(E(\frac{1}{n_c}\sum_{j=1}^{n_c} y_{c,i}|n_c))] \\ &= N_c^2 E(S_{cb}^2(\frac{1}{n_c} - \frac{1}{N_c})) + 0 \\ &= N_c^2 S_{cb}^2[E(\frac{1}{n_c}) - \frac{1}{N_c}]. \end{aligned} \tag{2.5}$$

(c) Since in the cluster sampling scheme, the clusters are selected under simple random sampling, according to Note 1, the desired result is obtained. □

About the estimator $\widehat{\tau}_c$ the following note is important.

Note 2. a) To obtain the variance of $\widehat{\tau}_c$, we need to calculate $E(\frac{1}{n_c})$, which is difficult in practice, but given the positive value for n_c, a good approximation of $E(\frac{1}{n_c})$ is as follows (Stephen 1945):

$$E(\frac{1}{n_c}) \simeq \frac{N}{nN_c} + \frac{N(N - N_c)}{n^2 N_c^2}. \tag{2.6}$$

b) If N_c is unknown, then $\widehat{\tau}_c$ is not an estimator for τ_c.

The following theorem introduces an estimator for τ_c when N_c is unknown.

Theorem 2.2. *a) Estimator*

$$\tilde{\tau}_c = \frac{N}{n}\sum_{i=1}^{n_c} y_{c,i},$$

is an unbiased estimator for τ_c.

b) The variance of this estimator is as follows:

$$V(\tilde{\tau}_c) = N^2 S_{cb}^{2*}[\frac{1}{n} - \frac{1}{N}].$$

If

$$Y_{ij}^* = \begin{cases} Y_{c,ij}, & U_{ij} \in c, \\ 0, & o.w. \end{cases} \qquad for(i = 1, \dots, N\ ;\ j = 1, \dots, M_i),$$

and $Y_i^* = \sum_{j=1}^{M_i} Y_{ij}^*$, then

$$S_{cb}^{2*} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i^* - \overline{Y}^*)^2,$$

where $\overline{Y}^*$ is the average of $Y_1^*, \dots, Y_N^*$. Obviously, the value of Y_i^* is zero for clusters that are empty from subpopulation members.

c)An unbiased estimator for variance of part b) is,

$$v(\tilde{\tau}_c) = s_{cb}^{2*}[\frac{1}{n} - \frac{1}{N}],$$

where if the sample units are u_{ij} $(i = 1, \dots, n\ ;\ j = 1, \dots, m_i)$,

$$y_{ij}^* = \begin{cases} y_{c,ij}, & u_{ij} \in c, \\ 0, & o.w. \end{cases} ,$$

and

$$y_i^* = \sum_{j=1}^{m_i} y_{ij}^*,$$

then

$$s_{cb}^{2*} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i^* - \overline{y}^*)^2,$$

and $\overline{y}^*$ is the average of $y_1^*, \dots, y_n^*$.

Proof. a) It can be easily shown that

$$\tilde{\tau}_c = \frac{N}{n}\sum_{i=1}^{n_c} y_{c,i} = \frac{N}{n}\sum_{i=1}^{n} y_i^*,$$

so

$$\begin{aligned} E(\tilde{\tau}_c) &= NE(\frac{1}{n}\sum_{i=1}^{n} y_i^*) \\ &= N(\frac{1}{N}\sum_{i=1}^{N} Y_i^*) \\ &= \sum_{i=1}^{N_c} Y_{c,i} = \tau_c. \end{aligned}$$

b)

$$\begin{aligned} V(\tilde{\tau}_c) &= N^2 V(\frac{1}{n}\sum_{i=1}^{n} Y_i^*) \\ &= N^2 S_{cb}^{*2}[\frac{1}{n} - \frac{1}{N}]. \end{aligned}$$

c) Since the clusters are selected by simple random sampling and according to Note 1, this part is easily established. □

Proposition 2.1. *The estimator* $\widehat{\mu}_c = \frac{\widehat{\tau}_c}{M_c}$ *with variance* $\frac{V(\widehat{\tau}_c)}{M_c^2}$ *is an unbiased estimator for* μ_c*, and also* $\widehat{\mu}_{cR} = \frac{\sum_{i=1}^{n_c} y_{c,i}}{\sum_{i=1}^{n_c} m_{c,i}}$ *is a biased estimator of* μ_c*, and an approximation value for the variance of this estimator is*

$$V(\widehat{\mu}_{cR}) \simeq \frac{E(\frac{1}{n_c}) - \frac{1}{N_c}}{\overline{M}_c^2(N_c - 1)}\sum_{i=1}^{N_c}(Y_{c,i} - M_{c,i}\mu_c)^2, \tag{2.7}$$

where $\overline{M}_c = \frac{1}{N_c}\sum_{i=1}^{N_c} M_{c,i}$.

By pasting $\overline{m}_c = \frac{1}{n_c}\sum_{i=1}^{n_c} m_{c,i}$ *and* $\widehat{N}_c = \frac{n_c}{n}N$*, respectively, as unbiased estimators of* $\overline{M}_c$ *and* N_c *in Equation (2.7), a good estimator of* $V(\widehat{\mu}_{cR})$ *is as follows:*

$$v(\widehat{\mu}_{cR}) \simeq \frac{\frac{1}{n_c} - \frac{n}{n_c N}}{\overline{m}_c^2 (n_c - 1)} \sum_{i=1}^{n_c} (y_{c,i} - m_{c,i}\widehat{\mu}_{cR})^2.$$

Examples 2.1. In 2016, a one-stage cluster sampling with 60 clusters was selected to estimate the total amount of cereal (wheat, barley, and maize) produced in South Khorasan province comprising 183 rural areas (clusters). Each farm in the selected villages in the sample was examined as a small unit. The sample is the weight of cereal in tons and is given in Table 1.

Table 1: Quantities of cereal produced at tonnes in clusters of sample

Cluster	Grain	Cluster	Grain	Cluster	Grain	Cluster	Grain
1	276	16	466	31	690	46	794
2	738	17	540	32	842	47	476
3	686	18	748	33	754	48	457
4	704	19	1049	34	654	49	685
5	529	20	730	35	794	50	563
6	919	21	714	36	408	51	448
7	911	22	625	37	685	52	555
8	527	23	637	38	693	53	706
9	460	24	316	39	304	54	469
10	259	25	564	40	421	55	787
11	1018	26	359	41	380	56	570
12	421	27	681	42	668	57	663
13	793	28	726	43	838	58	704
14	549	29	829	44	398	59	812
15	640	30	457	45	457	60	931

Based on this sample, the estimated total cereal production of the province is as follows:

$$\widehat{\tau} = \frac{183}{60}(276 + 738 + \cdots + 931) = 114245,$$

and the estimation of its variance is

$$v(\widehat{\tau}) = N^2 s_b^2 (\frac{1}{n} - \frac{1}{N}) = 12836085.$$

After estimating the total amount of cereal, we are interested in estimating the total amount of wheat and the total amount of barley produced in the province. Sample data for total wheat and barley at tonnes are given in Table 2.

Table 2: Wheat and barley quantities produced at tonnes in the sample

Cluster	Wheat	Barley	Cluster	Wheat	Barley	Cluster	Wheat	Barley
1	276	0	21	608	46	41	243	95
2	521	65	22	625	0	42	518	78
3	489	131	23	472	87	43	698	62
4	594	72	24	316	0	44	215	74
5	422	66	25	482	36	45	420	0
6	679	187	26	219	68	46	587	105
7	820	46	27	527	105	47	312	120
8	434	93	28	508	124	48	428	29
9	205	155	29	689	75	49	568	46
10	185	38	30	457	0	50	413	73
11	719	128	31	517	58	51	302	98
12	325	56	32	654	131	52	321	82
13	612	112	33	590	78	53	378	0
14	480	42	34	417	45	54	318	112
15	517	102	35	594	112	55	619	58
16	387	0	36	320	61	56	420	8
17	487	25	37	685	0	57	518	78
18	650	84	38	504	85	58	341	218
19	981	0	39	228	0	59	620	142
20	482	212	40	313	108	60	778	67

Based on the available information, we know that wheat was produced in all population clusters, so $N_c = N = 183$ and to estimate the total amount of province's wheat production, we use $\widehat{\tau}_c$,

$$\widehat{\tau}_c = \frac{183}{60}(276 + 521 + \cdots + 778) = 90057.$$

Hence n_c is a constant number and the estimation of variance of this estimator is

$$v(\widehat{\tau}_c) = N_c^2 s_{cb}^2 (\frac{1}{n_c} - \frac{1}{N_c}) = 10482468.$$

There is no information about the number of villages producing barley, so to estimate province's barley production, $\tilde{\tau}_c$ is used, and we have

$$\tilde{\tau}_c = \frac{183}{60}(65 + 131 + \cdots + 67) = 13575.$$

The estimation of the variance of this estimator is as follows:

$$v(\tilde{\tau}_c) = N^2 s_b^{*2} (\frac{1}{n} - \frac{1}{N}) = 988189.$$

One of the topics discussed in this section is the comparison of $MSE(\widehat{\mu}_{cR})$ and $V(\widehat{\mu}_c)$. Since n_c is a random variable, this is not theoretically feasible. But they can be compared using a hypothetical example.

3 Simulated Study

In this section, we conduct a simulation study to assess the performance of the proposed estimators. Consider a population with 100 clusters. We randomly select 100 numbers from 500 to 2000 and consider them as the size of clusters in this population. Suppose that in this population there are 40 clusters containing members of the sub-population so that the number and size of members of the sub-population of these clusters are randomly selected from 1-100 and 1- M_i, respectively, where M_i for $i = 1, 2, ..., 100$ is size of i-th cluster. We simulate the members of each cluster from Normal distribution with different means and variance equal to 1. Then we select samples with sizes ($n = 20, 40, 70$),repeated 1000 times, by simple random sampling method from the clusters of this population. Estimator $T = (\widehat{\tau}_c, \tilde{\tau}_c, \widehat{\mu}_c, \widehat{\mu}_{cR})'$ is obtained as follow:

$$T = \frac{1}{1000} \sum_{i=1}^{1000} T_i,$$

where T_i is the value of the estimator in the i-th iteration. Details of the values of the estimators and their variances are given in Table 3 and Table 4. It should be noted that $\tau_c = 147354.6$, $M_c = 29726$ and $\mu_c = 4.957$.

Table 3: the values of $\widehat{\tau}_c, \tilde{\tau}_c$ and their variances

	n	$\widehat{\tau}_c$	$\tilde{\tau}_c$	$V(\widehat{\tau}_c)$	$V(\tilde{\tau}_c)$
1	20	147237.9	146228.8	1760.04	2896.47
2	40	146653.5	146539.5	654.21	1184.59
3	70	147498.6	147659.6	187.88	326.48

Table 4: the values of $\widehat{\mu}_c, \widehat{\mu}_{cR}$ and their variances

	n	$\widehat{\mu}_c$	$\widehat{\mu}_{cR}$	$V(\widehat{\mu}_c)$	$V(\widehat{\mu}_{cR})$
1	20	4.953	5.000	1.991	1.212
2	40	4.933	4.964	0.740	0.444
3	70	4.961	4.961	0.212	0.124

According to Tables 3 and 4, it seems that estimator $\widehat{\tau}_c$ is more accurate than $\tilde{\tau}_c$, and estimator $\widehat{\mu}_{cR}$ performs better than $\widehat{\mu}_c$.

4 Conclusions

We introduced some estimators for the mean and total value subpopulation parameters under one-stage cluster sampling where the subpopulation size is known or unknown. The introduced estimators do not have a double cost because these estimators are based on a general sample taken from the population. In an example we also estimated the total value of wheat and barley in South Khorasan, in 2016, using a real data related to the amount of grain in this province. In addition, based on a simulation study, we observed the precision of the estimators and saw that $\widehat{\tau}_c$ and $\widehat{\mu}_{cR}$ are better estimators for τ_c and μ_c, respectively.

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