

On Exponential Power Distribution And Poultry Feeds Data: A Case Study

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Abstract. In this paper, we propose to study a generalized form of the exponential power distribution which contains others in the literature as special cases. This unifying exponential power distribution is characterized by a parameter ω and a function $h(\omega)$ which regulates the tail behavior of the distribution, thus making it more flexible and suitable for modeling than the usual normal distribution, while retaining symmetry. We derive several mathematical and statistical properties of this distribution and estimate the parameters using both the moments and maximum likelihood approach, obtaining the information matrix in the process. The multivariate extension of the distribution is also examined. Finally we fit the univariate generalized exponential power distribution as well as the normal distribution to data on eggs produced by chicken on each of two different poultry feeds (inorganic and organic copper-salt compositions) and show that the generalized exponential power distribution fit is considerably better. We then use the Kolmogorov-Smirnov two samples one-tailed test to show that there is an increase in egg weights and decrease in cholesterol level when the feed is organic.

Keywords. Exponential power distribution, information matrix, inorganic copper-salt, maximum likelihood estimation, multivariate exponential power distribution, organic copper-salt, poultry feeds data.

MSC: 60E05, 62H10, 6207.

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Received: June 2012; Accepted: November 2012

1 Introduction

The exponential power distribution is a class of densities which includes the normal and allows thick tails, Thus making it more suitable in modeling when compared with the usual normal distribution. In fact, it is a natural generalization of the normal distribution and first introduced by Subbottin (1923). Gomez et al. (1998) presented a multivariate version of the exponential power distribution and Lindsey (1999) used this distribution to model repeated measurements. We begin this section with a proposition.

Proposition 1.1. *let X be a random variable then,*

$$f(x; \mu, \sigma, \omega) = \frac{\omega h(\omega)}{2\Gamma(\frac{1}{\omega})\sigma} \exp \left\{ - \left[\frac{h(\omega) |x - \mu|}{\sigma} \right]^\omega \right\}$$

is a probability density function (p.d.f.) with three parameters $\omega > 0$, $\sigma > 0$, $\mu \in \mathfrak{R}$. The tail region is regulated by the function $h(\omega)$, which is positive for all $\omega > 0$.

Proof. Since

$$f(x; \mu, \sigma, \omega) = \int_{-\infty}^{\infty} \frac{\omega h(\omega)}{2\Gamma(\frac{1}{\omega})\sigma} \exp \left\{ - \left[\frac{h(\omega) |x - \mu|}{\sigma} \right]^\omega \right\} dx = 1, \quad (1)$$

where $\mu \in \mathbb{R}$ and $\sigma \in (0, \infty)$ are respectively, location and scale parameters and the function $h(\cdot)$ for all $\omega > 1$ is positive, the theorem is proved

Note that (1) reduce to: (i) Subbottin (1923) distribution if $h(\omega) = \frac{1}{\omega}$; and (ii) Univariate power exponential distribution studied by Gomez, et. al. (1998), when $\omega = 2\beta$ and $2h(\omega) = 1$. The function $h(\omega)$ regulates the tail region, hence the distribution is more flexible when compare to normal distribution.

Remark 1.1. We note that the density in equation (1), henceforth called a generalized form of the exponential power density (GEP), was developed from the data (see Appendix B) obtained in the experiment carried out on poultry feeds data. The model gave a better fit when compared with the normal distribution.

By symmetry, the odd central moments are zero. If the random variable X follows a GEP distribution we may write $Z = \left(\frac{X-\mu}{\sigma}\right)$. Then the n th moment of Z is

$$E(Z^n) = \frac{1 + (-1)^n}{2\Gamma(\frac{1}{\omega})} h(\omega)^{1-\frac{n+1}{\omega}} \Gamma\left(\frac{n+1}{\omega}\right). \quad (2)$$

Hence the k th moment of X is

$$E(X^k) = E[(\mu + \sigma Z)^k]$$

Now, using binomial expansion we have;

$$\sum_{k=0}^n \binom{n}{k} \mu^{n-k} \sigma^k E(Z^k) = \sum_{k=0}^n \mu^{n-k} \sigma^k \frac{1 + (-1)^k}{2\Gamma(\frac{1}{\omega})} (h(\omega))^{1-n} \Gamma(\frac{k+1}{\omega})$$

Proposition 1.2. *The following expressions for the mean, variance, and Kurtosis of a random variable $X \sim GEP(\mu, \sigma, \omega)$ hold:*

$$\begin{aligned} E(X) &= \mu; \\ E(X^2) &= \frac{1}{h(\omega)} \left\{ \mu^2 + \frac{\sigma^2 \Gamma(\frac{3}{\omega})}{\Gamma(\frac{1}{\omega})} \right\}; \\ E(X^3) &= \frac{1}{[h(\omega)]^2} \left\{ \mu^3 + \frac{3\mu\sigma^2 \Gamma(\frac{3}{\omega})}{\Gamma(\frac{1}{\omega})} \right\} \text{ and} \\ E(X^4) &= \frac{1}{[h(\omega)]^3} \left\{ \mu^4 + \frac{6\mu^2\sigma^2 \Gamma(\frac{3}{\omega})}{\Gamma(\frac{1}{\omega})} + \frac{\sigma^4 \Gamma(\frac{5}{\omega})}{\Gamma(\frac{1}{\omega})} \right\} \end{aligned}$$

Proof. The proof follows easily from the above expressions for the binomial expansion. The skewness is obviously zero as for the case of normal distribution.

2 Maximum Likelihood Estimation

Given a random sample of n observations x_1, \dots, x_n from the generalized exponential power distribution, then the log-likelihood function ℓ is:

$$\ell(\hat{\theta}) = n \ln \omega + n \ln h(\omega) - n \ln 2 - n \ln \Gamma(\frac{1}{\omega}) - n \ln \sigma - \sum_{i=1}^n \left[\frac{h(\omega) |x_i - \mu|}{\sigma} \right]^\omega$$

The likelihood equations for the three parameters are obtained by differentiating with respect to μ, σ and ω respectively and equating to zero. We have:

$$\frac{\partial \ell}{\partial \mu} = -\omega \left[\frac{h(\omega)}{\sigma} \right]^\omega \sum_{i=1}^n |x_i - \mu|^{\omega-1} \text{sgn}(x_i - \mu) = 0 \tag{3}$$

where

$$\text{sgn}(x_i - \mu) \equiv \begin{cases} 1, & \text{if } x_i > \mu \\ 0, & \text{if } x_i = \mu \\ -1, & \text{if } x_i < \mu \end{cases}$$

$$\begin{aligned}\frac{\partial \ell}{\partial \sigma} &= \frac{-n}{\sigma} + \omega^{-\omega-1} \sum_{i=1}^n \left[\frac{h(\omega) |x_i - \mu|}{\sigma} \right]^\omega \\ \hat{\sigma} &= h(\omega) \left(\frac{\omega}{n} \sum_{i=1}^n |x_i - \mu|^\omega \right)^{\frac{1}{\omega}},\end{aligned}\quad (4)$$

and hence

$$\begin{aligned}\frac{\partial \ell}{\partial \omega} &= \frac{n}{\omega} + \frac{nh'(\omega)}{h(\omega)} - \frac{n\Gamma'(1/\omega)}{\Gamma(1/\omega)} \\ &- \left\{ \left[\frac{h(\omega)}{\sigma} \right]^\omega \ln \left[\frac{h(\omega)}{\sigma} \right] + \frac{p[h(\omega)]^{\omega-1}}{\sigma^\omega} \right\} \sum_{i=1}^n |x_i - \mu|^\omega \\ &- \left[\frac{h(\omega)}{\sigma} \right]^\omega \sum_{i=1}^n |x_i - \mu|^\omega w_i(\mu) = 0\end{aligned}\quad (5)$$

where

$$w_i(\mu) \equiv \begin{cases} \ln |x_i - \mu|, & \text{if } x_i \neq \mu \\ 0, & \text{if otherwise} \end{cases}$$

The prime (') denote the differentiation with respect to ω . The joint likelihood equations for μ , σ and ω can be solved by simultaneously solving equations 3-5. The Fisher information matrix $I(\mu, \sigma, \omega)$ when $n=1$, has elements $-E [\partial^2 \ln f(x, \theta) / \partial \theta_i \partial \theta_j]$ for $i, j = 1, 2, 3$, where θ are the parameters under consideration in GEP distribution. The variance-covariance matrix $I^{-1}(\theta)$ of the ML estimates μ, σ, ω .

$$E \left(-\frac{\partial^2 \ln \ell}{\partial \mu^2} \right) = \frac{\omega(\omega-1)[h(\omega)]^2 \Gamma(1 - \frac{1}{\omega})}{\sigma^2 \Gamma(\frac{1}{\omega})} \quad (6)$$

$$E \left(-\frac{\partial^2 \ln \ell}{\partial \mu \partial \sigma} \right) = 0 \quad (7)$$

$$E \left(-\frac{\partial^2 \ln \ell}{\partial \mu \partial \omega} \right) = 0 \quad (8)$$

$$E \left(-\frac{\partial^2 \ln \ell}{\partial \sigma^2} \right) = \frac{\omega - n + 1}{\sigma^2} \quad (9)$$

$$E \left(-\frac{\partial^2 \ln \ell}{\partial \sigma \partial \omega} \right) = -\frac{1}{\sigma \omega} \left[1 + \frac{\omega^2 h'(\omega)}{h(\omega)} + \psi(1 + \frac{1}{\omega}) + \omega \ln h(\omega) \right] \quad (10)$$

and

$$E \left(-\frac{\partial^2 \ln \ell}{\partial \omega^2} \right) = \frac{n}{\omega^2} - \frac{nh(\omega)h''(\omega) - [h'(\omega)]^2}{[h(\omega)]^2}$$

$$\begin{aligned}
 & - \frac{n\Gamma(\frac{1}{\omega})\psi'(\frac{1}{\omega}) - [\psi(\frac{1}{\omega})]^2}{[\Gamma(\frac{1}{\omega})]^2} \\
 & + \omega \left[\frac{h'(\omega)}{h(\omega)} \right]^2 - h'(\omega) + \left[\frac{h'(\omega)}{h(\omega)} \right]^2 \\
 & - \frac{h'(\omega)}{h(\omega)} \left[\frac{\psi(1 + \frac{1}{\omega})}{\omega} + \ln h(\omega) \right] - \frac{h''(\omega)}{h(\omega)} - \frac{h'(\omega)}{\omega h(\omega)} \quad (11)
 \end{aligned}$$

from this matrix obviously the shape parameter ω is orthogonal to the location parameter μ , but is not orthogonal to the scale parameter, σ .

3 The Multivariate Extension

Several authors (e.g. Gomez et al., 1998 and Fang et al. 1990) have studied the multivariate extension to the usual exponential power distribution. The procedure in Fang, et al. (1990) may be applied to the standard univariate density function, $f(x^2)$ defined for (1) as;

$$f(x^2) = \frac{dF(x; \omega)}{dx} = \frac{\omega h(\omega)}{2\Gamma(\frac{1}{\omega})} \exp \left\{ -[h(\omega)x^2]^{\frac{\omega}{2}} \right\} \quad (12)$$

replacing x^2 in (12) by the Mahalanobis (1936) distance $\Delta_{\Sigma}^2(x, \mu)$, gives

$$dF(\mathbf{x}; \mu, \Sigma, \omega) = \frac{K d^n \mathbf{x}}{2\Gamma(\frac{1}{\omega})} [h(\omega)]^{1/2} \exp - [h(\omega)(\mathbf{x} - \mu)^T \Sigma^{-1}(\mathbf{x} - \mu)]^{\omega/2} \quad (13)$$

where K is the normalization constant is given by;

$$\sqrt{\pi^n |\Sigma|} \frac{\Gamma(\frac{n}{\omega})}{\Gamma(\frac{1}{\omega})\Gamma(\frac{n}{2})} [h(\omega)]^{(n-1)/2} \quad (14)$$

substituting in (13) we obtain

$$\begin{aligned}
 dF(\mathbf{x}; \mu, \Sigma, \omega) & = \frac{d^n \mathbf{x}}{\sqrt{\pi^n |\Sigma|}} \frac{\Gamma(1 + \frac{n}{\omega})}{\Gamma(1 + \frac{n}{\omega})} [h(\omega)]^{n/2} \\
 & \times \exp \left\{ - \left[h(\omega)(\mathbf{x} - \mu)^T \Sigma^{-1}(\mathbf{x} - \mu) \right]^{\omega/2} \right\} \quad (15)
 \end{aligned}$$

Definition 3.1. A random vector $X = (X_1, \dots, X_n)^T$, has a n -dimensional generalized exponential power distribution with parameters μ, Σ, ω parameters, where $\mu \in \mathbb{R}^n$, Σ is a $(n \times n)$ positive definite symmetric matrix, and $h(\omega) > 0$, given that $\omega \in (0, \infty)$, if its density is as in equation (15).

Note that ω and its function $h(\omega)$ determines the kurtosis. Thus, the correlation structure can be obtained directly from Σ in the usual way. When $\omega = 2$, we have multivariate normal distribution; when $\omega = 1/2$, a form of multivariate double exponential distribution; $\omega \rightarrow \infty$, a multivariate uniform distribution; $\omega < 1$, we have tail longer than the normal; and $\omega > 1$ we have tail shorter than normal distribution.

The function (15) belong to elliptically contoured (EC) families of distributions introduced by Kelker (1970) and well investigated by Cambanis et.al. (1981) and by Fang, et.al. (1990). The corresponding density generator of random vector \mathbf{X} which has distribution (15) and belong to $EC(\mu, \Sigma, g)$ is $g(t) = \exp\{-h(\omega)t^{\omega/2}\}$, since the function g satisfies the condition $\int_0^\infty t^{\frac{n}{2}-1}g(t)dt < \infty$.

Proposition 3.1. *Let $X \sim EP(\mu, \Sigma, \omega)$ where $\mu \in \mathbb{R}^k$ and $\Sigma \in \mathbb{R}^{d \times k}$ is positive definite with $r(\Sigma) = k$. Then X can be represented stochastically by $X = \mu + RA^T u^{(k)}$, where $u^{(k)}$ is a k -dimensional random vector uniformly distributed on the unit sphere in \mathbb{R}^k , A is a square matrix of order n such as $A^T A = \Sigma$ and R is a non-negative random variable being stochastically independent of $U^{(k)}$ and with density*

$$f_R(r) = \frac{1 + \frac{n}{2}}{2\sqrt{\pi^n}\Gamma(1 + \frac{n}{p})} [h(\omega)]^{\frac{n}{2}} \exp\{-[h(\omega)|r|^\omega]\}, \quad (0 < r < \infty) \quad (16)$$

Proof. From Cambanis et al.(1981) and Fang et al.(1990), it follows that if a k -dimensional random vector \mathbf{X} has elliptical density, then it can be decomposed into $\mathbf{x} = \mu + RA^T u^{(k)}$. In what follows, we assume that \mathbf{x} possesses a generalized exponential power distribution $h(\mathbf{x}^T \mathbf{x})$. Given any nonnegative measurable function $g(\cdot)$. The density generator $h_R(r)$, is derived by observing that

$$E[g(r)] = \int_0^\infty g[(x^T x)^{\frac{1}{2}}]h(x^T x)dx,$$

setting $r = y^{1/2}$, we have

$$E[g(r)] = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2})} \int_0^\infty g(y^{1/2})y^{n/2-1}h(y)dy,$$

which upon simplification gives;

$$E[g(r)] = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2})} \int_0^\infty g(r)r^{n-1}h(r^2)dr,$$

which yielded density generator (16). This stochastic representation allows for generation of random variable X from the generalized exponential power distribution by (a) determine a matrix A such that $A^T A = \Sigma$; (b) simulate a vector u^k uniformly distributed on the unit sphere; (c) generate an observation of a random variable with density (16); (d) put $X = \mu + RA^T u^{(k)}$. This can be implemented using codes written in R-software.

Remark 3.1. A random vector $X = (X_1; \dots; X_p)^T$ with values \mathbf{x} in R^p and having a Generalized Exponential Power (GEP) Distribution is said to belong to the family of elliptically symmetric distributions with location parameter $\mu = (\mu_1; \dots; \mu_p)^T \in \mathbb{R}^p$ and symmetric positive definite scale matrix Σ , if its probability density function (pdf), can be expressed as a function of the quadratic form $(\mathbf{x} - \mu)^T \Sigma^{-1} (\mathbf{x} - \mu)$ as follows:

$$f_X(\mathbf{x}) = |\Sigma|^{-1/2} g[(\mathbf{x} - \mu)^T \Sigma^{-1} (\mathbf{x} - \mu)] \tag{17}$$

where g is a function on $[0, \infty)$ satisfying $\int_{\mathbb{R}^p} g(y^T y) dy = 1$. Also by transformation, setting $Y = C^{-1}(x - \mu)$, a random vector $X = (X_1; \dots; X_p)^T$ from Generalized Exponential Power Distribution is said to have a distribution belonging to the family of spherically symmetric distributions if X and OX have the same distributions for all $p \times p$ orthogonal matrices O . where C is a $p \times p$ nonsingular matrix satisfying $\Sigma = C^T C$. The pdf of Y is given by $g(y^T y)$.

3.1 The Multivariate Test Statistics

In this section we propose a statistic that may be used to test the hypothesis that a given random vector is from a multivariate generalized exponential power distribution. Let $\{W_i^2\}_{i=1}^N$ represent an ordered set of sample values of $\Delta_{\Sigma}^2(\mathbf{x}, \mu)$. Let w^2 be the value of the distance metric, and denote the density function and distribution function of w^2 as $f'(w^2)$ and $F'(w^2)$ respectively. The probability that $W^2 < w^2$ where W^2 is a particular sample value of the distance metric, is equal to the probability that x_i lies within the region Ω_W defined as the hyper-volume enclosed by the hyper-ellipsoidal shell parameterized by particular value of the distance metric, i.e.,

$$\Pr(W^2 < w^2) = F'(w^2) = \Pr(\mathbf{x} \in \Omega_W),$$

where

$$W^2 = (x - \mu)^T \Sigma^{-1} (x - \mu).$$

Therefore,

$$F'(W^2) = \int \dots \int_{\Omega_W} K f \left\{ (\mathbf{x} - \mu)^T \Sigma^{-1} (\mathbf{x} - \mu) \right\} d^n x \quad (18)$$

Applying this in (15) and after some algebra we obtain;

$$F'(W^2) = \frac{\gamma \left[\frac{n}{\omega}, \{h(\omega)W^2\}^{\frac{\omega}{2}} \right]}{\Gamma(\frac{n}{\omega})}, \quad (19)$$

where $\gamma(\cdot)$ is the lower incomplete gamma function (see Gradshteyn and Ryzhik (2007, pg. 657)). To test the null hypothesis that a given dataset has distribution (15) we may use the Kolmogorov statistic

$$D_N = \max_i |S_i - F'(W_i^2)|, \quad (20)$$

where $\{S_i\}_{i=1}^N$ are the order statistics from the given sample.

The Kolmogorov-Smirnov test on the distance metric was preferred because it is easy to define, understand, and execute.

4 Application in Poultry Feeds Data

Cholesterol is a waxy substance that comes from two main sources: The liver and food intake. Stewart (2010, pp. 13) noted that too much cholesterol can clog the arteries, lower blood flow to other tissue in the body, thereby precipitate heart diseases. Eggs have often been frowned at because of high cholesterol contents, therefore causing decreased in consumption. But Majumder and Wu (2009) note that eggs are inexpensive source of high-quality protein and other nutrients. It has also widely reported that eggs contain lecithin and phospholipids, integral to the construction of brain cell membrane. In terms of feeding intellect, their value lies mainly in the quality of their proteins. Long used as points of reference when analyzing the quality of other dietary proteins by the Food and Agriculture Organisation (FAO), eggs are actually rich in amino acids, essential in the production of the principal neurotransmitters. Hence, instead of a total boycott, it is better to reduce the risk associated with eggs consumption.

In order to alleviate the problem associated with consumption, an organic copper-salt combination was used instead of the inorganic combinations currently used in preparation of poultry feeds. 96 chickens were randomly selected and randomly divided into two groups of 48

each. Each group was subjected to the same general conditions and treatments, differing only in that while, the chickens in the first group were fed with inorganic copper-salt, those in the second group were fed with inorganic copper-salt. After 4 Months the weight(gram) and cholesterol level(mg/egg) of the eggs yielded by the two groups were measured. The data are presented in the Appendix B.

Normality assumptions are common in data analysis, but a close look at the normal and generalized exponential power Q-Q plot in Figures 1 and 2 of Appendix A, though by the normal distribution is a reasonable fit, the tails seem to be shorter than expected and hence p-values resulting from the usual tests based on normal assumptions cannot be trusted. On the other hand, the generalized exponential power distribution proves to be a much better fit for egg weights as well as cholesterol level in both groups. We now carry out estimation of the parameters of the distribution using the method of moments and maximum likelihood estimate (MLE). These methods were preferred because they have many optimal properties in estimation: sufficiency; consistency; efficiency; and parameterization invariance which are rarely found in other approaches (For details on MLE see Spanos (1999) and Casella and Berger (2002)). The estimated values from the observations namely the means, standard deviation (s.d.), the ω , Akaike information Criterion (AIC) as well as Bayesian Information criterion (BIC) are given in Table 1 with the corresponding log-likelihood ($\log(\ell)$) estimates. Since the explicit expression cannot be obtained for μ and ω in the estimation of maximum likelihood in the Section 2 we employed a numerical approach using a written code in the R software programme. The statistical computing environment (Ihaka & Gentleman, 1996), was supplemented with the package called "normp" downloaded into R file from the site <http://cran.r-project.org/> and used to analyze the data. As earlier stated, though the normal distribution was a good fit to the sample data, but we have a better fit when we use the generalized exponential power distribution. This is evident when comparing the results from the log-likelihood functions, AIC and BIC given in Table 1 below, as well as the plots in figures 1 and 2.

Table 1: Parameters Estimation for eggs weights in Appendix B

density	Variables	mean	s.d	shape(ω)	$h(\omega)$	$\log(\ell)$	AIC	BIC
normal	weight (inorganic)	58.350	3.559	mil	nil	-129.055	262.11	273.5948
	weight (organic)	59.10	1.822	mil	mil	-96.910	197.82	209.3048
	cholesterol (inorganic)	195.728	21.907	mil	nil	-216.282	436.564	448.0488
	cholesterol (organic)	131.457	37.232	mil	nil	-241.739	487.478	498.9628
GEP	weight (inorganic)	58.507	4.371	4.470	0.281	-126.028	258.056	275.2832
	weight (organic)	59.118	2.306	5.963	0.168	-91.850	189.700	206.9272
	cholesterol (inorganic)	194.481	28.359	6.825	0.147	-210.809	427.618	444.8452
	cholesterol (organic)	129.720	46.327	5.206	0.192	-237.448	480.896	498.1232

4.1 Hypothesis Testing

We have used a numerical algorithm written in the R software environment to find estimates of the parameters giving the best fit for the data in appendix B. These estimates are reported in Table 1. We used the Kolmogorov-Smirnov two-sample test for large samples to decide whether there is significant difference in the weights and cholesterol levels of the two groups. Clearly, if we find that there is a significant difference, then feeds should be changed to organic copper-salts. Note that we have used a one sided Kolmogorov-Smirnov two-sample test i.e. the test compares the cumulative frequency distributions of the two samples and determines whether the observed D indicates that they have been drawn from different populations, one of which is stochastically larger than the other. Let $F_{n_1}(X)$ be the cumulative step function of the sample observations for the inorganic copper-salt type and let $F_{n_2}(Y)$ be the cumulative step function of the sample observations for organic copper-salt type. We test the null hypothesis that the two samples have been drawn from the same population against the alternative hypothesis that the values of the population from which one the samples was drawn are stochastically larger than the values of the population from which the other sample was drawn. In other words, for the eggs weights the null and alternative hypotheses are of the form $H_0 : F_{n_1}(X) = F_{n_2}(Y)$ and $H_A : F_{n_1}(X) < F_{n_2}(Y)$; also for the cholesterol levels while the null hypothesis remain the same, the alternative hypothesis is $H_A : F_{n_2}(X) < F_{n_1}(Y)$. We now define

$$D = \text{maximum} [F_{n_1}(X) - F_{n_2}(Y)]$$

The sampling distribution of D is assumed to be a generalized exponential power distribution. It has been shown by Goodman (1954) that

$$\chi_2^2 = 4D^2 \frac{n}{2} \quad (21)$$

(for number of observations are the same)

1. for the egg weights $D = 0.3125$ with corresponding $P[\chi_2^2 > 9.21] < 0.01$. Hence, we reject H_0 and conclude that the weight of the eggs fed with inorganic copper salt is less than the organic type.
2. for the cholesterol level which is the most important, we have: $D = 0.6875$ with $P[\chi_2^2 > 45.375] < 0.0001$. Hence we reject the the null hypothesis and conclude that, using the organic copper-salt type the cholesterol level significantly reduced.

5 Concluding Remarks

We have proposed a generalized exponential power distribution, and studied some of its mathematical and statistical properties. We fitted this distribution to data arising from an experiment concerning the cholesterol level and weight of eggs. The Q-Q plots clearly show that the generalized exponential power distribution fits the data better than the usual normal distribution. Finally, hypotheses tests show that consumption of eggs from chicken fed with organic copper-salt should not be boycotted for the fear of high cholesterol level. This assertion is further buttressed with the box plots in Figure 4 of Appendix A. Therefore we recommend that the constituents of poultry feeds should change from the inorganic to organic combinations.

Acknowledgments

I wish to express my sincere gratitude to the Authorities and Managements of Universita di Padova, Italy and the COIMBBRA group of universities for the opportunities availed to me during a three months fellowship at the University where part of this research was carried out under the advice of Prof. Adelchi Azzalini. I also thank Prof. Matthew Ojo of the Obafemi Awolowo University, Ile-Ife, Nigeria for his meaningful contributions. The very useful comments by the anonymous referee significantly improved the paper.

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Appendix A

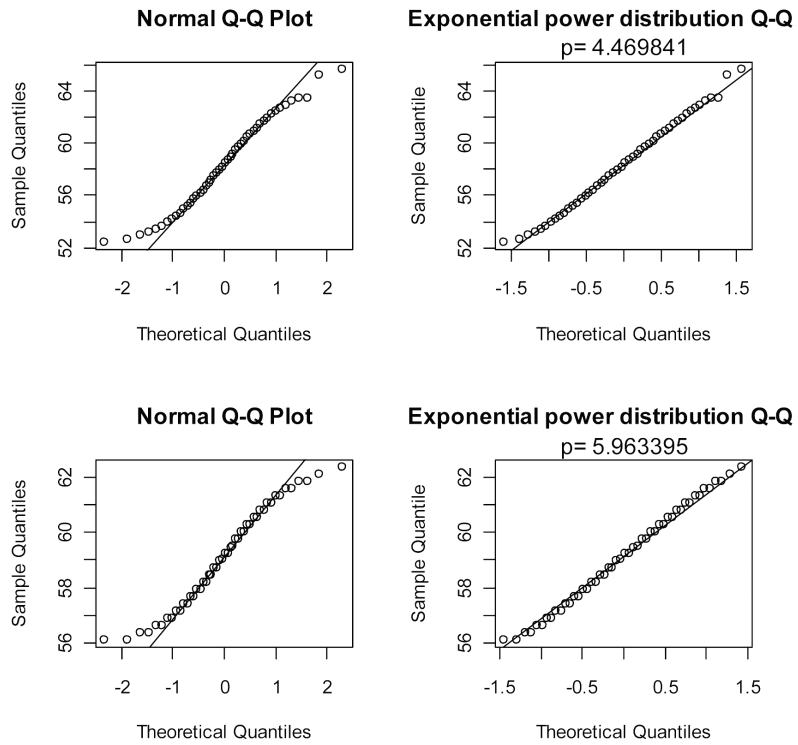


Fig.1: Top row are the Q-Q plots weight of the eggs for the inorganic copper-salt, the first is the normal and the second is the Generalized Exponential distribution. The second row is the case of organic copper-salt .

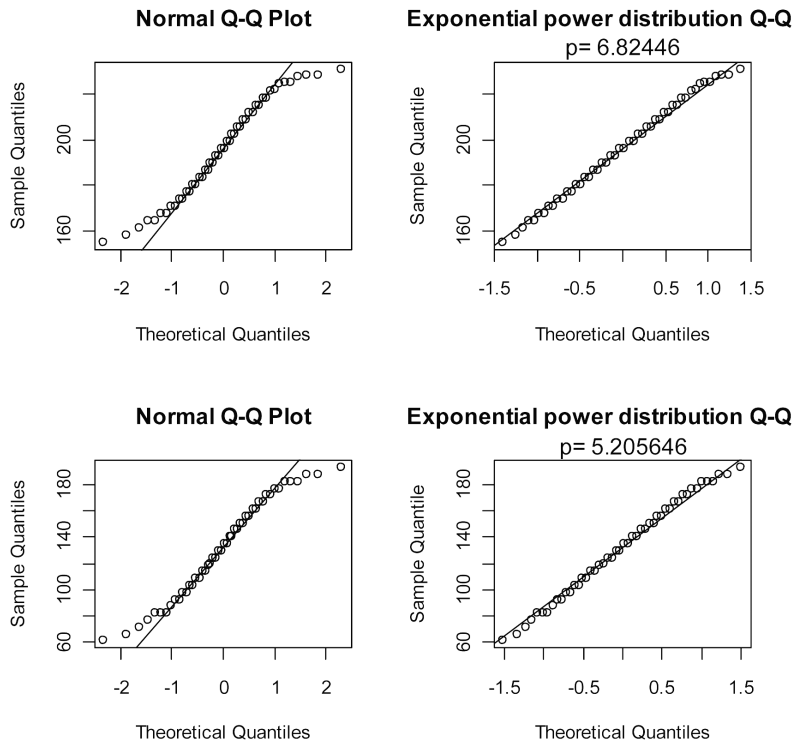


Fig. 2: The Q-Q plots for the egg cholesterol content. The first row is the inorganic copper-salt for normal and Generalized Exponential Power Distribution. The second row is the organic copper-salt.

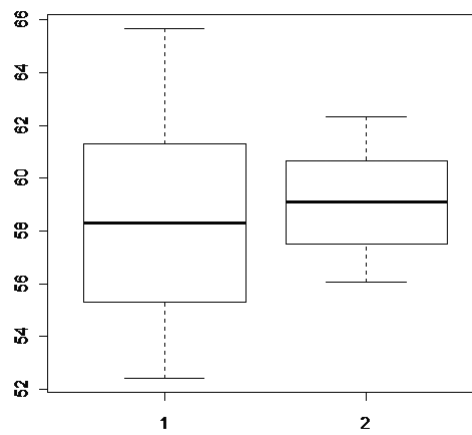


Figure 3: Box plot of eggs weight for inorganic(1) and Organic (2) copper-salt combination

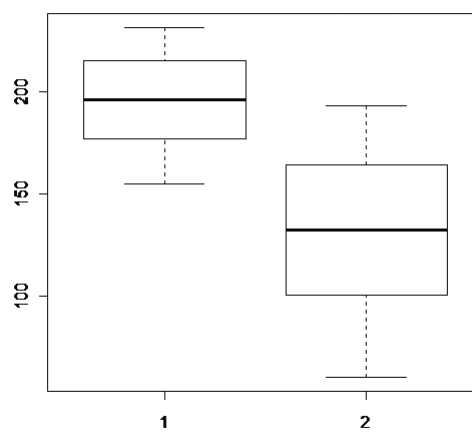


Figure 4: Box plot of eggs cholesterol contents for inorganic(1) and Organic (2) copper-salt combinations

Appendix B

Figure 5: Observed Data from Inorganic and Organic Copper-Salt

Inorganic copper-salt		Organic copper-salt	
weight	cholesterol	weight	cholesterol
52.67	164.23	56.08	56.08
53.17	167.42	56.34	56.34
53.67	170.6	56.61	56.61
54.17	173.78	56.87	56.87
54.67	176.96	57.13	57.13
55.17	180.14	57.39	57.39
55.67	183.32	57.65	57.65
56.17	186.51	57.92	57.92
56.67	189.69	58.18	58.18
57.17	192.87	58.44	58.44
57.67	196.05	58.7	58.7
58.17	199.24	58.96	58.96
58.67	202.42	59.23	59.23
59.17	205.6	59.45	59.45
59.67	208.78	59.75	59.75
60.17	211.96	60.01	60.01
60.67	215.14	60.27	60.27
61.17	218.33	60.54	60.54
61.67	221.52	60.8	60.8
62.17	224.69	61.06	61.06
62.67	224.85	61.32	61.32
63.17	227.88	61.58	61.58
63.43	228.03	61.85	61.85
65.67	231.06	62.34	62.34
65.15	228.01	62.11	62.11
63.43	224.83	61.85	61.85
62.93	221.65	61.58	61.58
62.43	218.46	61.32	61.32
61.93	215.28	61.06	61.06
61.43	212.1	60.8	60.8
60.93	208.92	60.54	60.54
60.43	205.74	60.27	60.27
59.93	202.56	60.01	60.01
59.43	199.37	59.75	59.75
58.93	196.19	59.49	59.49
58.43	193.01	59.23	59.23
57.93	189.83	59	59
57.43	186.65	58.7	58.7
56.93	183.46	58.44	58.44
56.43	180.28	58.18	58.18
55.93	177.1	57.92	57.92
55.43	173.72	57.65	57.65
54.93	170.74	57.39	57.39
54.43	167.55	57.13	57.13
53.93	164.37	56.87	56.87