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Economic Design of $T^2 - VSSC$ Chart Using Genetic Algorithms

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Abstract. The principal function of a control chart is to help management distinguish different sources of variation in a process. Control charts are widely used as a graphical tool to monitor a process in order to improve the quality of the product. Chen and Hsieh (2007) have designed a T^2 control chart using a Variable Sampling Size and Control limits ($VSSC$) scheme. They have shown that using the $VSSC$ scheme results in charts with more statistical power to detect small to moderate shifts in the process mean vector than the other T^2 charts. In this paper, we develop an economic design for the $T^2 - VSSC$ chart to help determine the design parameters and then minimize the cost model proposed by Costa and Rahim (2001) using a Genetic Algorithm (GA) approach. We also compare economic design of the $T^2 - VSSC$ chart with the $T^2 - DWL$, $T^2 - VSSI$ and $T^2 - FRS$ charts so as to choose the best option and, finally, carry out a sensitivity analysis to investigate the effects of model parameters on the solution of the economic design.

Keywords. Adjusted average time to signal ($AATS$), economic design (ED), genetic algorithm (GA), Markov chain, multivariate control charts, sensitivity analysis, variable sample size and control limits ($VSSC$).

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1 Introduction

An effective method for improving the quality of productions or services in companies is Statistical Process Control (SPC). Control charts are the most common tools of SPC for detecting the occurrence of an assignable cause which leads to non-conforming products and hence a change mean and variance values from target values. In some industrial situations, quality control problems may characterize a single characteristic which is a normal continuous random variable. When process quality can be described by a single quality characteristic, univariate control charts are used to maintain current control of the process. The most common control chart for monitoring the mean of a single variable is the $\bar{X}$ chart. Walter Shewhart created these charts in the 1920s to monitor processes to detect any large shift in process mean and process variance. However, it is increasingly common today for processes to be characterized by more than one variable which are usually correlated and have a multivariate normal distribution. For example, resistance of an industrial piece (e.g. a wheelwork), its dimensions and its weight have univariate normal distribution, separately; But the correlation between these characteristics is not zero and hence, joint distribution of them is multivariate normal. First time, Hotelling (1947) has developed quality control procedures for several related random variables (or process characteristics) with normal distribution. Among these procedures, Hotelling's T^2 control chart is probably the most widely known and applied in industry. Of course, recently Mason and Young (2002) showed that if correlated quality characteristics have non-normal distribution, the distribution of T^2 will be a function of Beta distribution which is not studied in this paper.

Generally, the traditional practice for applying a control chart to monitor a process is to obtain samples of fixed size at fixed sampling intervals between successive samples. This procedure is called Fixed Ratio Sampling (FRS). The efficiency of FRS scheme is good for large shifts but not for small or even moderate shifts. The use of the $\bar{X} - FRS$ control chart requires the user to select three design parameters: the sample size (n), the sampling interval (h), and the width of the control limit (k). To improve the efficiency, the FRS policy was modified to a Variable Ratio Sampling (VRS) policy. One procedure for comparing the statistical performance of VRS schemes is a Variable Sample Size (VSS) scheme. In this scheme, the sample size varies and is a function of prior sample results. The use of the $\bar{X} - VSS$ control chart requires the user to select five design parameters: the small and large sample sizes (n_1, n_2), the sampling interval (h), the warning limit (w), and the control limit (k). The statistical design of the VSS scheme for univariate Shewhart control chart has been studied by Burr (1969), Daudin (1992), Prabhu et al (1993), Costa (1994) and Zimmer et al (1998). Aparisi (1996) generalized VSS scheme to the multivariate case and Faraz and Moghadam (2008) compared the $T^2 - VSS$ chart with the $T^2 - FRS$ chart. Another procedure is the Variable Sampling Interval (VSI) scheme where the sampling interval is a function of prior sample results. The $\bar{X} - VSI$ chart was introduced by Reynolds (1988), Reynolds and Arnold (1989) and Runger and Pignatiellow (1991). Also, the $T^2 - VSI$ chart was studied by Aparisi and Haro (2001) and Faraz et al (2009a). Variable Sample size and Sampling Interval ($VSSI$) is another procedure in VRS schemes where both sample size and sampling interval are functions of prior sample results. Prabhu et al (1994) were the first to study the $\bar{X} - VSSI$ chart. Costa (1997) obtained similar results when comparing the $VSSI$ scheme with the VSS, VSI and FRS schemes. Aparisi and Haro (2003) developed $VSSI$ scheme to the multivariate case. The Double Warning Line (DWL) scheme designed by Faraz and Parsian (2006) for the multivariate case, is also a procedure in VRS schemes where there are two warning lines and both, sample size and sampling interval, are functions of prior sample results.

As control charts became ubiquitous in industrial practice, researchers became concerned over the economic consequences of control charts design and control charts which were designed based on statistical criteria and cost parameters in a process became less popular. The method of designing control charts based on economic models is called Economic Design (ED). Based on the ED procedure, the charts are designed in such a way that the overall costs associated with maintaining current control of a process are minimized. Duncan (1956) proposed the first economic model and used it to ED of the Shewhart $\bar{X}$ chart. Also, Lorenzen and Vance (1986) developed a cost model which is appropriate for all kind of control charts. Costa and Rahim (2001), proposing a new economic model made a comparison between the FRS and Variable Parameters (VP) schemes in the univariate case. Montgomery and Klatt (1972) were the first to design a multivariate FRS control chart, economically. Chou et al (2006) studied the ED of the $T^2 - VSI$ control chart. Also, Chen (2006, a-b) studied ED of $T^2 - VSI$ and $T^2 - VSSI$ charts. Faraz et al (2009b) and Faraz et al (2010a) economically designed the $T^2 - VSS$ chart using the Costa and Rahim (2001) and Lorenzen and Vance (1986) economic models. Also, Faraz et al (2010b) studied the

ED of the $T^2 - DWL$ chart using the Costa and Rahim (2001) economic model.

Chen and Hsieh (2007) designed a new VRS scheme called Variable Sample Size and Control limits ($VSSC$), to improve the power of T^2 control charts, statistically. In some industries, economic aspects are more important than statistical aspects. So, in this paper, we investigate the effect of incorporating the $T^2 - VSSC$ chart into economic designs and compare the ED of the $T^2 - VSSC$ chart with the other VRS schemes. Also, for investigating the effects of model parameters on the economic model, sensitivity analysis is used. The paper is organized as follows: In Section 2, the $T^2 - VSSC$ chart and a Markov chain approach to $VSSC$ scheme are briefly reviewed. In Section 3, the cost model proposed by Costa and Rahim (2001) is used to build a model of process controlled by the $T^2 - VSSC$ chart. In Section 4, the Genetic Algorithm (GA) is employed to obtain the optimal values of the parameters. Also, the $VSSC$ scheme is compared with other VRS schemes. In Section 5, a sensitivity analysis is carried out to investigate the effects of model parameters on the solution of the economic design and finally, section 6 contains a conclusion.

2 $T^2 - VSSC$ Chart and the Markov Chain Approach

Consider a process in which p correlated quality characteristics are measured simultaneously and the distribution of these quality characteristics is a p-variate normal with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. In practice, the vector $\boldsymbol{\mu}$ and matrix $\boldsymbol{\Sigma}$ are usually unknown and estimated using the sample mean vector, $\bar{\mathbf{X}}$, and the sample variance-covariance matrix, $\bar{\mathbf{S}}$.

$$\hat{\boldsymbol{\mu}} = \bar{\mathbf{X}} = \frac{1}{m}\sum_{i=1}^{m}\bar{\mathbf{X}}_i \ , \qquad \bar{\mathbf{X}}_i = \frac{1}{n}\sum_{j=1}^{n}\mathbf{X}_{ij} \ ,$$

$$\hat{\boldsymbol{\Sigma}} = \bar{\mathbf{S}} = \frac{1}{m}\sum_{i=1}^{m}(\bar{\mathbf{X}}_i - \bar{\mathbf{X}})(\bar{\mathbf{X}}_i - \bar{\mathbf{X}})'$$

where $\mathbf{X}_{ij}$ is the jth sample in the ith subgroup and $\bar{\mathbf{X}}_i$ is the mean of the ith subgroup. Moreover, n is the sample size for all subgroups and m is number of subgroups in the initial sampling from the process. When the vector $\boldsymbol{\mu}$ and matrix $\boldsymbol{\Sigma}$ are known, the charting statistic, for every subgroup, is $T^2 = n(\bar{\mathbf{X}} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\bar{\mathbf{X}} - \boldsymbol{\mu})$ which is plotted on a control chart. The vector $\boldsymbol{\mu}_0' = (\mu_{01}, \ldots, \mu_{0p})$ is the vector of in-control means for quality characteristics. Assuming to unknown $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, the ith subgroup statistic, $T_i^2 = n(\bar{\mathbf{X}}_i - \bar{\mathbf{X}})'\boldsymbol{S}^{-1}(\bar{\mathbf{X}}_i - \bar{\mathbf{X}})$ for $i = 1, 2, \ldots, m$, is plotted on a control chart in sequential order. If a value of this statistic is greater than the Upper Control Limit (UCL), the process will be considered out of control. Otherwise, the process is in control.

In statistical design methodology, if the process parameters ($\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$) are known, T^2 has a chi-square distribution with p degrees of freedom and so $UCL = \chi^2_{(p,\alpha)}$. In practical situations, $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are unknown and for each i, the distribution of T_i^2 is the Hotelling distribution and $UCL = C(m, n, p)F(p, \nu, \alpha)$, where

$$C(m,n,p) = \begin{cases} \dfrac{p(m+1)(n-1)}{m(n-1)-p+1} & n > 1 \\ \\ \dfrac{p(m+1)(m-1)}{m(m-p)} & n = 1 \end{cases} \qquad and$$

$$\nu = \begin{cases} m(n-1)-p+1 & n > 1 \\ \\ m-p & n = 1 \end{cases}$$

It is usually assumed that the variance-covariance matrix is fixed but unknown and an assignable cause occurs upon a change of the mean from $\boldsymbol{\mu}_0$ to $\boldsymbol{\mu}_1$. The magnitude of this shift is expressed by Mahalanobis distance, $d^2 = (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)$. If the process is out of control ($d \neq 0$), the chart statistic will be distributed as a non-central distribution with non-centrality parameter $\eta = nd^2$.

In the FRS scheme, the chart parameters (sample size, n_0, sampling interval, h_0, and UCL) are fixed. Applying this method is very simple but its efficiency in detecting the small or moderate shifts is not good enough and hence, VRS schemes are necessary. One VRS scheme is the $VSSC$ scheme in which we have a fixed sampling interval, h, two sample sizes, n_1 and n_2 ($n_1 < n_2$), two warning lines, w_1 and w_2 ($w_1 > w_2$) and two control limits, k_1 and k_2 ($k_1 > k_2$). The sample size and sampling interval in a subgroup depend totally on the appearance of a prior sample point on the chart. We describe this chart below.

If the sample point falls in the interval $[0, w_i]$, $i = 1, 2$, the next sample size will be n_1 and the warning line and control limit for the next sample will be w_1 and k_1, respectively. When the sample point falls in the interval $(w_i, k_i]$, $i = 1, 2$, the next sample size will be n_2 and the warning line and control limit for the next sample will be w_2 and k_2, respectively. Therefore, the $T^2 - VSSC$ chart is defined as follows:

$$LCL = 0$$

$$UCL = C(m, n_j, p)F(p, \nu, 1-\alpha)$$

$$(n_i, w_i, k_i) = \begin{cases} (n_1, w_1, k_1) & 0 < T^2_{i-1} \le w_j \\ \\ (n_2, w_2, k_2) & w_j < T^2_{i-1} \le k_j \end{cases} , \qquad j = 1, 2$$

If $T^2_{i-1} > k_j$, we say the process is out of control. However, if there is not any assignable cause, then the signal is a false alarm. Note that the sample size at the start of the process is chosen at random.

In this chart, at each sampling stage, one of the six following states may occur according to the status of the process.
State 1: $0 < T^2 \le w_i$ and the process is in control $(d = 0)$.
State 2: $w_i < T^2 \le k_i$ and the process is in control $(d = 0)$.
State 3: $T^2 > k_i$ and the process is in control $(d = 0)$.
State 4: $0 < T^2 \le w_i$ and the process is out of control $(d \neq 0)$.
State 5: $w_i < T^2 \le k_i$ and the process is out of control $(d \neq 0)$.
State 6: $T^2 > k_i$ and the process is out of control $(d \neq 0)$.

The control chart produces a signal when $T^2 > k_i$. If the signal is genuine, then the process should be stopped and after repair, it starts to work again. The signal in state 3 is a false alarm and the signal in state 6 is a genuine alarm (absorbing state in Markov chain). The transition matrix between states may be written as

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ 0 & 0 & 0 & p_{44} & p_{45} & p_{46} \\ 0 & 0 & 0 & p_{54} & p_{55} & p_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where p_{ij} denotes the transition probability from state i to state j. We have defined p_{ij}'s as follows:

$$
\begin{aligned}
p_{11} &= P(0 < T^2 \leq w_1)e^{-\lambda h} = F(\frac{w_1}{C(m,n_1,p)}, p, \nu_1, 0) \times e^{-\lambda h} \\
p_{12} &= P(w_1 < T^2 \leq k_1)e^{-\lambda h} \\
&= [F(\frac{k_1}{C(m,n_1,p)}, p, \nu_1, 0) - F(\frac{w_1}{C(m,n_1,p)}, p, \nu_1, 0)] \times e^{-\lambda h} \\
p_{13} &= P(T^2 > k_1)e^{-\lambda h} = [1 - F(\frac{k_1}{C(m,n_1,p)}, p, \nu_1, 0)] \times e^{-\lambda h} \\
p_{14} &= P(0 < T^2 \leq w_1)(1 - e^{-\lambda h}) \\
&= F(\frac{w_1}{C(m,n_1,p)}, p, \nu_1, 0) \times (1 - e^{-\lambda h}) \\
p_{15} &= P(w_1 < T^2 \leq k_1)(1 - e^{-\lambda h}) \\
&= [F(\frac{k_1}{C(m,n_1,p)}, p, \nu_1, 0) - F(\frac{w_1}{C(m,n_1,p)}, p, \nu_1, 0)] \times (1 - e^{-\lambda h}) \\
p_{16} &= P(T^2 > k_1)(1 - e^{-\lambda h}) \\
&= [1 - F(\frac{k_1}{C(m,n_1,p)}, p, \nu_1, 0)] \times (1 - e^{-\lambda h}) \\
p_{21} &= p_{31} = P(0 < T^2 \leq w_2)e^{-\lambda h} = F(\frac{w_2}{C(m,n_2,p)}, p, \nu_2, 0) \times e^{-\lambda h} \\
p_{22} &= p_{32} = P(w_2 < T^2 \leq k_2)e^{-\lambda h} \\
&= [F(\frac{k_2}{C(m,n_2,p)}, p, \nu_2, 0) - F(\frac{w_2}{C(m,n_2,p)}, p, \nu_2, 0)] \times e^{-\lambda h} \\
p_{23} &= p_{33} = P(T^2 > k_2)e^{-\lambda h} = [1 - F(\frac{k_2}{C(m,n_2,p)}, p, \nu_2, 0)] \times e^{-\lambda h} \\
p_{24} &= p_{34} = P(0 < T^2 \leq w_2)(1 - e^{-\lambda h}) \\
&= F(\frac{w_2}{C(m,n_2,p)}, p, \nu_2, 0) \times (1 - e^{-\lambda h}) \\
p_{25} &= p_{35} = P(w_2 < T^2 \leq k_2)(1 - e^{-\lambda h}) \\
&= [F(\frac{k_2}{C(m,n_2,p)}, p, \nu_2, 0) - F(\frac{w_2}{C(m,n_2,p)}, p, \nu_2, 0)] \times (1 - e^{-\lambda h}) \\
p_{26} &= p_{36} = P(T^2 > k_2)(1 - e^{-\lambda h}) \\
&= [1 - F(\frac{k_2}{C(m,n_2,p)}, p, \nu_2, 0)] \times (1 - e^{-\lambda h}) \\
p_{44} &= P(0 < T^2 \leq w_1) = F(\frac{w_1}{C(m,n_1,p)}, p, \nu_1, \eta_1) \\
p_{45} &= P(w_1 < T^2 \leq k_1) \\
&= F(\frac{k_1}{C(m,n_1,p)}, p, \nu_1, \eta_1) - F(\frac{w_1}{C(m,n_1,p)}, p, \nu_1, \eta_1) \\
p_{46} &= P(T^2 > k_1) = 1 - F(\frac{k_1}{C(m,n_1,p)}, p, \nu_1, \eta_1)
\end{aligned}
$$

$$\begin{aligned}
p_{54} &= P(0 < T^2 \leq w_2) = F(\frac{w_2}{C(m,n_2,p)}, p, \nu_2, \eta_2) \\
p_{55} &= P(w_2 < T^2 \leq k_2) \\
&= F(\frac{k_2}{C(m,n_2,p)}, p, \nu_2, \eta_2) - F(\frac{w_2}{C(m,n_2,p)}, p, \nu_2, \eta_2) \\
p_{56} &= P(T^2 > k_2) = 1 - F(\frac{k_2}{C(m,n_2,p)}, p, \nu_2, \eta_2),
\end{aligned}$$

where $F(x,p,\nu_i,\eta_i)$ is defined as the cumulative probability distribution function of a non-central F distribution with p and $\nu_i = m(n_i-1) - p + 1$ degrees of freedom and non-centrality parameter $\eta_i = n_i d^2$, and $C(m,n_i,p) = \frac{p(m+1)(n_i-1)}{m(n_i-1)-p+1}$. When the mean vector and variance-covariance matrix are known, instead of $F(x,p,\nu_i,\eta_i)$ we use $F(x,p,\eta_i)$ which is defined as the cumulative probability distribution function of a non-central χ^2 distribution with p degrees of freedom and non-centrality parameter η_i.

In the investigations of Costa (1994), Duncan (1956), Lorenzen and Vance (1986), Faraz and Moghadam (2008), Montgomery and Klatte (1972), Chou et al (2006), the most recently used statistical measure to compare the efficiency of different control schemes, is $AATS$, the average time from the process mean shift until the chart produces a signal. This statistical measure determines the speed with which a control chart detects a process mean shift and is related to the average time of the cycle (ATC) which is the average time from the start of the production until the production of first signal after the process shift. If it is assumed that the shift in the process mean occurs at some random time in the future (not at beginning) and that this random time has an exponentially distributed random variable with mean $\frac{1}{\lambda}$, then the time that the process is in control, has an exponential distribution with parameter λ. So, the average time before occurrence of assignable cause is $\frac{1}{\lambda}$ and the steady-state $AATS$ is

$$AATS = ATC - \frac{1}{\lambda}.$$

We can compute the average time of the cycle using the Markov chain property (see Cinlar, 1975).

$$ATC = \mathbf{b}'(I-Q)^{-1}\mathbf{h}$$

where $\mathbf{b}' = (p_1, p_2, p_3, 0, 0)$ is a vector of initial probabilities with $\sum_{i=1}^{3} p_i = 1$, I is the identity matrix of order 5, Q is the 5×5 matrix obtained from P by deleting the absorbing row and column and $\mathbf{h}' = (h, h, h, h, h)$. Also $\mathbf{b}'(I - Q)^{-1}$ provides the expected number of trials needed to reach the absorbing state. In this paper, we choose $\mathbf{b}' = (0, 1, 0, 0, 0)$ for extra protection and for preventing problems that may be encountered during start-up.

We may also calculate the expected number of false alarm (ANF), the expected number of inspected items (ANI) and the expected number of samples (ANS) as follows:

$$\begin{aligned} ANF &= \mathbf{b}'(I - Q)^{-1}(0, 0, 1, 0, 0)' \\ ANS &= \mathbf{b}'(I - Q)^{-1}(1, 1, 1, 1, 1)' \\ ANI &= \mathbf{b}'(I - Q)^{-1}(n_1, n_2, n_2, n_1, n_2)' \end{aligned}$$

3 The Economic Model and Optimization

In this paper, we apply Costa and Rahim's cost model (2001) to study the ED of a $T^2 - VSSC$ chart. To formulate an economic model for the design of a control chart, it is necessary to make certain assumptions about the behavior of the process.

Suppose that the p quality characteristics follow a p-variate normal distribution with mean vector $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$. At the beginning the process is in control. In this state, we have $\boldsymbol{\mu} = \boldsymbol{\mu}_0$ and only an assignable cause changes the process mean from $\boldsymbol{\mu} = \boldsymbol{\mu}_0$ to $\boldsymbol{\mu} = \boldsymbol{\mu}_1$ ($\boldsymbol{\mu}_1$ is known). Also the variance-covariance matrix $\boldsymbol{\Sigma}$ stays constant. The assignable cause occurs according to a Poisson process with a mean of λ occurrences per an hour. So, the time interval that the process remains in control is an exponential variable with mean $\frac{1}{\lambda}$. Also, the process is not self-correcting. Hence, the process can be returned to the in-control state from an out-of-control state only by management intervention and appropriate corrective actions. We assume that the quality cycle starts in an in-control state and continues until the process reaches an out-of-control state, when it is repaired. The quality cycle follows a renewal reward process. Note that during the search for an assignable cause, the process is shut down.

According to the model, the production cycle may be divided to four time intervals: An in-control process period, a period of searching for a false alarm, an out-of-control period and a period for identifying the assignable cause and correcting the process. It is clear that the expected time the process is in control, is equal to $\frac{1}{\lambda}$. The expected out-of-control time period is the expected time from the process mean shift until an out-of-control signal is triggered. This expected time is given by $AATS$ and ATC is the average time from the moment of stating process to the time of chart signal after the process shift. Let T_0 be the average time wasted searching for an assignable cause when the process is in control and T_1 be the average time to find and correct the assignable cause. Hence, the expected time of a production cycle is given by

$$\begin{aligned} E(T) &= \frac{1}{\lambda} + T_0 ANF + AATS + T_1 \\ &= ATC + T_0 ANF + T_1 \end{aligned}$$

The profit from a production cycle includes the average profit while the process is in control as well as where it is out of control. We also incur a cost when searching for false alarms or assignable causes, for repairing the process and for sampling and inspecting items. So, the expected net profit from a production cycle is given by

$$E(I) = V_0 \frac{1}{\lambda} + V_1 AATS - C_0 ANF - C_1 - sANI$$

where V_0 is the average profit per hour earned when the process is in control, V_1 is the average profit per hour earned when the process is out of control, C_0 is the expected cost of searching for false alarms, C_1 is the expected cost of searching for an assignable cause and repairing the process and s is the cost of inspecting an item. Due to the renewal reward assumption, the expected net profit per hour is $\frac{E(I)}{E(T)}$.
Hence, the loss function, $E(L)$, is given by

$$E(L) = V_0 - \frac{E(I)}{E(T)}$$

To obtain the ED of a $T^2 - VSSC$ chart, we must obtain optimal values for the seven chart parameters $(k_1, k_2, w_1, w_2, n_1, n_2, h)$ of the cost model, E(L), given the process parameters $(p, \lambda, d, T_0, T_1)$ and the cost parameters (V_0, V_1, C_0, C_1, s). Among the seven chart parameters, the sample sizes are discrete variables (where $1 \leq n_1 < n_0 < n_2$) and the other variables are continuous (where $0 < w_1 < k_1$, $0 < w_2 < k_2$, $0 < w_2 < w_1$ and $0 < k_2 < k_1$). The maximum value of the parameter h is considered as the maximum hours available in a work shift, i.e. $h \leq 10$. Therefore, the general optimization problem is defined as follows:

$$\begin{aligned} &Min\ E(L) \\ &subject\ to: \quad && 0 < w_1 < k_1 \\ &&& 0 < w_2 < k_2 \\ &&& 0 < w_2 < w_1 \\ &&& 0 < k_2 < k_1 \\ &&& 0.1 \leq h \leq 10 \\ &&& 1 \leq n_1 < n_0 < n_2 \\ &&& n_1, n_2 \in \mathbb{Z}^+. \end{aligned}$$

This model is a non-linear function of the chart parameters with mixed continuous-discrete variables and a discontinuous and non-convex solution space. So, linear methods are inefficient for the minimization problem and non-linear programming techniques to search for optimal solution are necessary.

4 A Numerical Comparison between $T^2 - FRS$, $T^2 - VSSI$, $T^2 - DWL$ and $T^2 - VSSC$ Control Charts

For a fair comparison of the economic performance of the $T^2 - VSSC$ chart with other T^2 control charts (DWL, $VSSI$ and FRS), they should have the same costs when the process is in control. Two schemes which have the same in-control time, are comparable if and only if they have the same in-control cycle cost i.e. they must have the same expected number of false alarms, the same expected number of inspected items and the same expected number of samples. On the other hand, ANF, ANS and ANI values in all of these schemes should be equal during the in-control period. Here, for the sake of simplicity, we suppose that the vector $\boldsymbol{\mu}$ and matrix $\boldsymbol{\Sigma}$ are known. Hence, for the FRS scheme, the in-control statistical measures are

$$ANF = \frac{\alpha e^{-\lambda h_0}}{1 - e^{-\lambda h_0}}\ , \qquad ANS = \frac{1}{1 - e^{-\lambda h_0}}\ , \qquad ANI = \frac{n_0}{1 - e^{-\lambda h_0}}$$

and in the $VSSC$ scheme, these measures are

$$ANF = \frac{BF(w_2,p,0)e^{-\lambda h} + A(1-F(k_2,p,0))e^{-\lambda h}}{A[1-(1-F(w_2,p,0))e^{-\lambda h}]},$$

$$ANS = \frac{F(w_2,p,0)e^{-\lambda h}[1-(F(w_1,p,0)-F(w_2,p,0))e^{-\lambda h}+A]}{A[1-(1-F(w_2,p,0))e^{-\lambda h}]},$$

$$ANI = \frac{F(w_2,p,0)e^{-\lambda h}\{n_1[1-(1-F(w_2,p,0))e^{-\lambda h}]+n_2[(1-F(w_1,p,0))e^{-\lambda h}+A]\}}{A[1-(1-F(w_2,p,0))e^{-\lambda h}]}.$$

where

$$\begin{aligned} A &= 1-[1-F(w_2,p,0)+F(w_1,p,0) \\ &\quad -(F(w_1,p,0)-F(w_2,p,0))e^{-\lambda h}]e^{-\lambda h} \\ B &= e^{-\lambda h}\{(1-F(w_1,p,0))(1-F(k_2,p,0))e^{-\lambda h} \\ &\quad +(1-F(k_1,p,0))[1-(1-F(w_2,p,0))e^{-\lambda h}]\} \end{aligned}$$

Equating the statistical measures in these two schemes, we have $h = h_0$ and

$$k_2 = F^{-1}(1-K,p,0)$$

where

$$K = \frac{[1-(1-F(w_2,p,0))e^{-\lambda h}]\{\alpha e^{-\lambda h_0}A-(1-e^{-\lambda h_0})e^{-2\lambda h}F(w_2,p,0)(1-F(k_1,p,0))\}}{(1-e^{-\lambda h_0})e^{-\lambda h}[F(w_2,p,0)(1-F(w_1,p,0))e^{-2\lambda h}+A]}$$

and

$$n_2 = \frac{n_0\{A[1-(1-F(w_2,p,0))e^{-\lambda h}]\}-n_1F(w_2,p,0)e^{-\lambda h}[1-(1-F(w_2,p,0))e^{-\lambda h}](1-e^{-\lambda h_0})}{F(w_2,p,0)e^{-\lambda h}[(1-F(w_1,p,0))e^{-\lambda h}+A](1-e^{-\lambda h_0})}$$

Note that for $i = 1,2$, $F(x,p,\eta_i)$ is defined as the cumulative probability distribution function of a non-central chi-square distribution with p degrees of freedom and non-centrality parameter $\eta_i = n_i d^2$.

Therefore, choosing optimal values of k_1, w_1, w_2 and fixing n_1, n_0 and h_0, we can obtain k_2 and n_2. This optimization problem is a decision problem with continuous decision variables and a discontinuous and non-convex solution space and may be solved using Meta heuristic methods, such as Genetic Algorithms (GAs)(Goldberg, 1989).

In this paper, for a given parameters (k_0, n_0, h_0), GA finds optimal values of k_1, w_1 and w_2 and then computes the values of k_2 and n_2 and, finally, E(L). The initial values of the used GA parameters are the crossover rate (r_C), the population sizes (N_{pop}) and the mutation rate (r_M) were set to $N_{pop} = 100$, $r_C = 0.2$ and $r_M = 0.25$. The number of iterations was set to 200.

The MATLAB/GA computer program is used to perform a comparison between these schemes using a numerical example. Consider a soft drink fabrication process involving two quality characteristics of interest. One of the quality characteristics is the pressure inside the soft drink bottle and the other is the gas volume present in the drink. These quality characteristics directly affect the product quality. Difference levels of the parameters were determined based on previous studies investigated which are given in Table 1.

As in Chen (2006a), Table 2 provides a Taguchi orthogonal array $L_{16}(2^9 4^2)$ of a mixed 24 levels experimental design for assigning 11 variables to the columns of the $L_{16}(2^9 4^2)$ orthogonal array. In the $L_{16}(2^9 4^2)$ orthogonal array experiment, there are 16 trials (16 different level combinations of the eleven variables).

To compare the effectiveness of the $VSSC$ and DWL, $VSSI$ and FRS schemes, we use the first level of the parameters in Table 1 for $d = 0.25(0.25)4$ and obtain the optimal values of the design parameters using GAs. For $d = 0.25(0.25)4$, tables 4, 5 and 6 respectively show the optimal design parameters and the expected hourly loss for the $T^2 - VSSC$ chart together with the outputs of the $T^2 - FRS$, $T^2 - DWL$ and $T^2 - VSSI$ control charts. For each trial in Table 2, the optimal solution to the design parameters is obtained by GAs with the objective of minimizing the expected hourly loss. Table 3 displays the optimal design parameters and the expected hourly loss for the $T^2 - VSSC$ control chart of these 16 trials. All comparisons ($VSSC$ and DWL, $VSSC$ and $VSSI$, and $VSSC$ and FRS) are fair. The results indicate that the expected hourly loss of the $VSSC$ scheme is smaller than the DWL, $VSSI$ and FRS schemes. In fact, for small to large values of d ($d = 0.25(0.25)4$), when compared with the FRS scheme, there an average hourly saving of approximately 4.05%. Also, a comparison of $VSSC$ and DWL schemes, shows an average hourly saving of 23.90% and a comparison of $VSSC$ and $VSSI$ schemes, an average hourly saving of 14.45%. For small to moderate shifts ($d < 2$), these amounts change to 17.36%, 19.76% and 14.08% respectively. The results are plotted in Figure 1. This figure shows that for small to moderate shifts (Figure 1 (a)), the $VSSC$ scheme has a greater economic advantage versus the DWL, $VSSI$ and FRS schemes. This is not unexpected. Faraz et al (2010b) concluded that the $T^2 - DWL$ control chart is more economical than the $T^2 - VSSI$, $T^2 - VSS$, $T^2 - VSI$ and $T^2 - FRS$ charts. So, we can conclude that, specially for small to moderate shifts, the $VSSC$ scheme is more economical than other VRS and FRS schemes.

5 Sensitivity Analysis

In this section, we conduct a sensitivity analysis for the above example to study the effects of the model parameters (process parameters and cost parameters) on the economic design of the $T^2 - VSSC$ chart. This study is carried out using experimental design and linear regression analysis. In each linear regression model, given values of process parameters $(p, \lambda, d, T_0, T_1, m)$ and given values of cost parameters (V_0, V_1, C_0, C_1, s) are regarded as possible independent variables while the test parameters and the expected total cost are treated as the dependent variables. The eleven model parameters considered in the sensitivity analysis and their corresponding level plans are shown in Table 1. In our plan we have nine parameters with two levels and two parameters with four levels.

To study the effect of the model parameters on the solution of economic design of the $VSSC$ scheme based on the data in Table 6, the statistical software SPSS is used to run the regression analysis for each dependent variable. For each dependent variable, the output of SPSS includes an ANOVA table for regression and a table of regression coefficients, showing the corresponding information about statistical hypothesis testing. Table 7 is the SPSS output for the small sample size (n_1). From the ANOVA in Table 7(a), we conclude that at least one model parameter significantly affects the value of small sample size. On examining Table 7(b), we find that both the parameters d and C_1 significantly affect the value of sample size n_1. We note that the sign of the coefficient of d and C_1 is negative which indicates that higher values of the magnitude of the shift and the expected cost of searching for an assignable cause reduce the sample size n_1. Table 8 is based on the large sample size (n_2) and shows that the parameters d and V_0 significantly affect the value of sample size n_2. Increasing values of the magnitude of the shift cause a decrease in the sample size n_2 and lower values of the average profit per hour earned when the process is in control leads to larger values for n_2.

Tables 9 and 10 are the SPSS outputs for the warning lines (w_1, w_2). From these tables, we see that the only model parameter that significantly affects the warning lines is p. Higher values of the number of correlated quality characteristics leads to larger values for w_1 and w_2. Tables 11 and 12 which display the results for changing control limits (k_1, k_2), show that both parameters d and p significantly affect the control limits. Higher values of the magnitude of the shift and number of correlated quality characteristics lead to larger values for k_1 and k_2. Table 13 gives the SPSS output for the expected loss value E(L). It is obvious that the parameters λ, V_0, T_1, d and m significantly affect the expected loss. Since a higher penalty cost of defective products leads to a higher total cost, to reduce the total cost, the penalty cost of defective products should be decreased as much as possible.

6 Conclusion

In the present paper, the economic design of the $T^2 - VSSC$ chart is developed based on the cost model proposed by Costa and Rahim (2001). The expected total cost per hour is minimized using GA and finally, the $T^2 - VSSC$ chart and the $T^2 - DWL$, $T^2 - VSSI$ and $T^2 - FRS$ control charts are compared from an economic viewpoint. Also, sensitivity analysis is carried out to investigate the effects of model parameters on the solution of the economic design. The results indicate that among the four possible schemes, the expected hourly loss of the $VSSC$ scheme is smaller than the DWL, $VSSI$ and FRS schemes, specially for small to moderate shifts. These results are presented on a graph. Since Faraz et al (2010b) concluded that the $T^2 - DWL$ control chart is more economical when compared with $T^2 - VSSI$, $T^2 - VSS$, $T^2 - VSI$ and $T^2 - FRS$ control charts, we conclude that the $VSSC$ scheme is more economical than the other VRS and FRS schemes.
Sensitivity analysis shows that C_1, s, V_1 and T_0 have no significant impact on the optimal solution of the $VSSC$ scheme, while the eight parameters m, s, d, C_0, V_0, T_1, λ and p play significant role in influencing the response parameters.

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Tables

Table 1. Levels for each model parameter

Model parameter	Level 1	Level 2	Level 3	Level 4
m	10	50	100	1000
d	0.5	1.0	1.5	2.0
s	5	10		
C_0	250	500		
C_1	50	500		
V_0	250	500		
V_1	50	100		
T_0	2.5	5		
T_1	1	10		
λ	0.01	0.05		
p	2	5		

Table 2. Experimental layout of the $L_{16}(2^9 4^2)$ array

No.	m	d	s	C_0	C_1	V_0	V_1	T_0	T_1	λ	p
1	10	0.5	5	250	50	250	50	2.5	1	0.01	2
2	10	1.0	5	250	50	500	100	5	10	0.05	5
3	50	0.5	5	500	500	250	50	5	10	0.05	5
4	50	1.0	5	500	500	500	100	2.5	1	0.01	2
5	50	1.5	10	250	50	250	100	2.5	1	0.05	5
6	50	2.0	10	250	50	500	50	5	10	0.01	2
7	10	1.5	10	500	500	250	100	5	10	0.01	2
8	10	2.0	10	500	500	500	50	2.5	1	0.05	5
9	100	0.5	10	250	500	500	100	2.5	10	0.01	5
10	100	1.0	10	250	500	250	50	5	1	0.05	2
11	1000	0.5	10	500	50	500	100	5	1	0.05	2
12	1000	1.0	10	500	50	250	50	2.5	10	0.01	5
13	1000	1.5	5	250	500	500	50	2.5	10	0.05	2
14	1000	2.0	5	250	500	250	100	5	1	0.01	5
15	100	1.5	5	500	50	500	50	5	1	0.01	5
16	100	2.0	5	500	50	250	100	2.5	10	0.05	2

Table 3. Optimal set of parameters for ED of the T^2-VSSC control chart for 16 trails

No.	n_1	n_2	w_1	w_2	k_1	k_2	$E(L)_{VSSC}$
1	14	21	2.20	1.70	5.73	4.63	40.71
2	12	18	8.10	7.60	15.18	14.18	211.20
3	6	22	3.63	3.13	5.05	4.75	146.28
4	9	16	4.30	3.80	10.70	8.85	42.96
5	4	8	7.02	6.52	12.95	12.72	54.74
6	3	5	1.91	1.40	12.10	11.85	71.32
7	4	7	2.49	1.99	10.02	9.54	43.72
8	3	6	6.21	5.71	16.67	16.50	107.25
9	3	25	3.17	2.68	8.30	7.44	122.18
10	5	9	2.58	2.08	7.22	6.33	90.80
11	15	23	2.29	1.79	5.46	4.85	176.69
12	8	12	6.77	6.27	12.34	11.07	52.29
13	4	7	2.47	1.97	10.53	9.85	212.08
14	4	6	4.87	4.37	18.47	18.30	20.01
15	7	12	7.74	7.24	18.90	18.41	34.31
16	2	5	2.54	2.04	11.15	11.00	95.96

* Minimum values of E(L) for given and optimal parameters

Table 4. Optimal set of parameters for ED of T^2-VSSC and T^2-FRS control charts

d	n_1	n_2	w_1	w_2	k_1	k_2	$E(L)_{VSSC}$	k_0	h_0	n_0	$E(L)_{FRS}$	% of improvement
0.25	2	5	1.07	0.57	1.88	1.60	58.34*	1.67	10	3	58.43	0.15
0.50	13	23	2.27	1.77	5.65	4.68	40.71*	5.21	5.96	16	48.71	16.42
0.75	10	17	2.63	2.13	7.51	6.41	31.02*	7.09	4.49	12	38.55	19.53
1.00	7	12	3.57	3.07	8.72	7.38	25.06*	8.36	3.65	8	31.98	21.64
1.25	5	10	3.61	3.11	9.51	8.66	21.43*	9.32	3.08	8	27.48	22.02
1.50	4	7	2.59	2.09	10.36	9.64	19.14*	10.10	2.68	5	24.23	21.01
1.75	3	5	1.89	1.39	11.16	10.37	17.25*	10.75	3.37	4	21.76	20.73
2.00	2	4	1.96	1.45	11.40	3.60	39.23	11.31	2.13	3	19.83*	-97.83
2.25	2	4	1.97	1.46	11.90	10.73	15.66*	11.80	1.94	3	18.27	14.29
2.50	2	4	1.97	1.47	12.30	11.34	14.83*	12.23	1.78	3	16.98	12.66
2.75	2	59	15.96	4.60	19.39	5.10	19.12	12.83	1.76	3	15.93*	-20.03
3.00	2	6	3.18	2.65	14.82	11.80	14.10*	13.54	1.80	3	15.18	7.11
3.25	2	20	6.62	5.26	15.26	10.20	14.09*	14.29	1.83	3	14.65	3.82
3.50	2	4	1.97	1.47	18.40	13.76	13.09*	15.08	1.85	3	14.29	8.40
3.75	2	4	1.97	1.46	18.75	14.89	12.75*	19.92	1.88	3	14.03	9.12
4.00	2	56	17.95	10.50	17.97	11.00	13.07*	16.81	1.88	3	13.87	5.77

* Minimum values of E(L) for given and optimal parameters

Table 5. Optimal set of parameters for ED of the T^2-DWL control chart

d	n_1	n_2	h_1	h_2	w_N	w_T	k	$E(L)_{DWL}$	$E(L)_{VSSC}$	% of improvement
0.25	2	4	10.00	9.00	1.05	1.55	1.67	63.39	58.34*	7.97
0.50	13	20	6.09	5.70	1.95	2.45	5.21	49.17	40.71*	17.21
0.75	11	18	4.59	3.79	4.45	4.95	7.09	38.85	31.02*	20.15
1.00	7	14	3.75	2.86	4.57	5.07	8.36	31.89	25.06*	21.42
1.25	5	15	3.18	0.19	5.40	5.90	9.31	27.74	21.43*	22.75
1.50	4	9	2.10	2.78	3.67	4.17	10.10	25.11	19.14*	23.77
1.75	3	16	2.47	0.89	5.92	6.42	10.76	23.01	17.25*	25.03
2.00	2	8	2.23	1.43	3.99	4.49	5.21	43.69	39.23*	10.21
2.25	2	16	2.04	0.33	5.96	6.46	11.31	20.73	15.66*	24.46
2.50	2	11	1.88	0.72	4.83	5.33	11.83	20.68	14.83*	28.29
2.75	2	4	1.88	1.50	1.89	2.39	12.87	20.59	19.12*	7.14
3.00	2	13	1.90	0.51	5.31	5.81	13.62	19.40	14.10*	27.32
3.25	2	5	1.93	1.49	2.57	3.07	14.26	19.31	14.09*	27.03
3.50	2	6	1.95	1.39	3.14	3.64	15.20	19.03	13.09*	31.21
3.75	2	12	1.98	0.71	5.10	5.60	16.22	18.58	12.75*	31.38
4.00	2	7	1.98	1.30	3.56	4.06	7.03	30.46	13.07*	57.09

* Minimum values of E(L) for given and optimal parameters

Table 6. Optimal set of parameters for ED of the $T^2 - VSSI$ control chart

d	n_1	n_2	h_1	h_2	w	k	$E(L)_{VSSI}$	$E(L)_{VSSC}$	% of improvement
0.25	2	4	10.00	9.00	0.50	1.67	59.55	58.34*	2.03
0.50	11	18	7.60	4.01	1.30	5.21	48.02	40.71*	15.22
0.75	8	13	6.57	2.06	1.30	7.09	37.19	31.02*	16.59
1.00	4	9	5.80	0.87	1.18	8.36	29.99	25.06*	16.44
1.25	2	7	4.65	1.06	1.19	9.31	25.75	21.43*	16.78
1.50	2	6	4.07	1.01	1.25	10.10	23.07	19.14*	17.03
1.75	2	5	3.96	0.14	1.09	10.76	20.17	17.25*	14.48
2.00	2	19	5.03	1.39	3.34	5.21	35.44*	39.23	-10.69
2.25	1	4	2.66	1.13	1.30	11.31	18.23	15.66*	14.10
2.50	1	4	2.40	1.07	1.29	11.83	17.85	14.83*	16.92
2.75	2	5	1.97	1.60	1.72	12.87	17.32*	19.12	-10.39
3.00	2	5	2.06	1.60	1.70	13.63	16.87	14.10*	16.42
3.25	2	5	2.00	1.70	1.75	14.26	16.54	14.09*	14.81
3.50	2	5	2.05	1.70	1.73	15.20	16.30	13.09*	19.69
3.75	2	5	2.11	1.70	1.72	16.22	16.13	12.75*	20.95
4.00	2	17	3.14	1.65	3.90	7.03	26.61	13.07*	50.88

* Minimum values of E(L) for given and optimal parameters

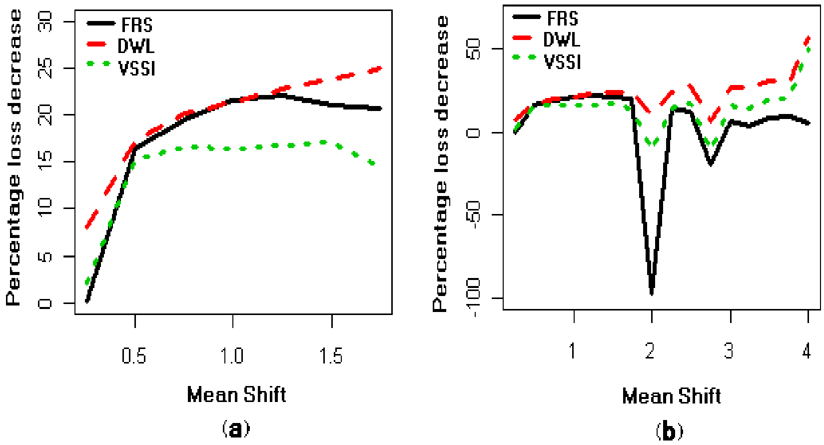

Figure 1: Percentage loss decrease of E(L) for the VSSC scheme versus FRS, DWL and VSSI schemes as a function of d.

Table 7(a). ANOVA table for sample size n_1

Source	Sum of Squares	DF	Mean Square	F Value	$P-Value$
Regression*	153.675	2	76.837	10.166	0.002
Residual	98.262	13	7.559		
Total	251.937	15			

* Predictors: (Constant), d, C_1

Table 7(b). Coefficients table for sample size n_1

Model	$\hat{\beta}$	Std. Error	t	$P-Value$
Constant	14.313	1.882	7.607	0.000
d	-4.650	1.230	-3.782	0.002
C_1	-0.008	0.003	-2.455	0.029

Table 8(a). ANOVA table for sample size n_2

Source	Sum of Squares	DF	Mean Square	F Value	$P-Value$
Regression*	680.050	2	340.025	54.104	0.000
Residual	81.700	13	6.285		
Total	761.750	15			

* Predictors: (Constant), d, V_0

Table 8(b). Coefficients table for sample size n_2

Model	$\hat{\beta}$	Std. Error	t	$P-Value$
Constant	22.750	2.427	9.373	0.000
d	-11.400	1.121	-10.168	0.000
V_0	0.011	0.005	2.194	0.047

Table 9(a). ANOVA table for warning line w_1

Source	Sum of Squares	DF	Mean Square	F Value	$P-Value$
Regression*	44.656	1	44.656	22.616	0.000
Residual	27.643	14	1.974		
Total	72.299	15			

* Predictors: (Constant), p

Table 9(b). Coefficients table for warning line w_1

Model	$\hat{\beta}$	Std. Error	t	$P-Value$
Constant	0.370	0.899	0.415	0.685
p	1.114	0.234	4.756	0.000

Table 10(a). ANOVA table for warning line w_2

Source	Sum of Squares	DF	Mean Square	F Value	$P-Value$
Regression*	44.723	1	44.723	22.684	0.000
Residual	27.601	14	1.972		
Total	72.324	15			

* Predictors: (Constant), p

Table 10(b). Coefficients for warning line w_2

Model	$\hat{\beta}$	Std. Error	t	$P-Value$
Constant	-0.113	0.891	-0.149	0.884
p	1.115	0.234	4.763	0.000

Table 11(a). ANOVA table for control limit k_1

Source	Sum of Squares	DF	Mean Square	F Value	$P-Value$
Regression*	223.524	2	111.762	21.870	0.000
Residual	66.433	13	5.110		
Total	289.957	15			

* Predictors: (Constant), d, p

Table 11(b). Coefficients table for control limit k_1

Model	$\hat{\beta}$	Std. Error	t	$P-Value$
Constant	-0.581	1.912	-0.304	0.766
d	5.425	1.01	5.367	0.0020
p	1.456	0.377	3.865	0.002

Table 12(a). ANOVA table for control limit k_2

Source	Sum of Squares	DF	Mean Square	F Value	$P-Value$
Regression*	255.768	2	127.884	30.986	0.000
Residual	53.653	13	4.127		
Total	309.421	15			

* Predictors: (Constant), d, p

Table 12(b). Coefficients table for control limit k_2

Model	$\hat{\beta}$	Std. Error	t	$P-Value$
Constant	-2.024	1.718	-1.178	0.260
d	5.892	0.909	6.485	0.000
p	1.511	0.339	4.463	0.001

Table 13(a). ANOVA table for $E(L)$

Source	Sum of Squares	DF	Mean Square	F Value	$P-Value$
Regression*	55889.275	5	11177.855	35.729	0.000
Residual	3128.506	10	312.851		
Total	59017.781	15			

* Predictors: (Constant), λ, V_0, T_1, d, m

Table 13(b). Coefficients table for $E(L)$

Model	$\hat{\beta}$	Std. Error	t	$P-Value$
Constant	-47.055	19.397	-2.426	0.036
λ	2085.937	221.095	9.435	0.000
V_0	0.217	0.035	6.127	0.000
T_1	5.383	0.983	5.478	0.000
d	-31.318	7.910	-3.959	0.003
m	0.027	0.011	2.532	0.030

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