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A Conditional Test for Exponentiality Against Weibull DFR Alternatives Based on Censored Samples

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Abstract. A conditional test based on quadratic form using type-2 censored sample for testing exponentiality against Weibull alternative is proposed. The simulated percentage points and powers are given. The proposed test performs well for identifying Weibull DFR alternative even for small sample. An example is also given.

1 Introduction

Weibull distribution is quite popular as a life testing model and for many other applications where a skewed distribution is required. Probably the main justification for consideration of the Weibull distribution is that it has been shown experimentally to provide a good

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fit for many different types of characteristics derived in an analysis of breaking strengths. This model is quite flexible and has the advantage of having a closed form of cumulative distribution function (CDF).

A two parameter Weibull distribution is defined by the pdf

$$f(x;\theta,\beta) = \frac{\beta}{\theta} x^{\beta-1} e^{-x^{\beta}/\theta}, x > 0, \theta, \beta > 0. \tag{1.1}$$

where β is the shape parameter and θ is the scale parameter. This model includes the exponential distribution with constant failure rate (CFR) for $\beta = 1$ and provide an increasing failure rate (IFR) for $\beta > 1$ and decreasing failure rate (DFR) for $\beta < 1$. Hence test for β is of interest, because it could indicate whether simple exponential model is adequate or whether more general Weibull DFR (or IFR) model should be used.

Thoman et. al (1969) have considered the problem of testing of hypothesis regarding the shape parameter based on complete samples. Bain and Engelhardt (1986) have proposed a modified version of Thoman et. al (1969) test statistic whose asymptotic distribution is approximated to a chi-squared distribution. For the complete inferences of Weibull distribution under different censoring schemes, one may refer to Bain and Engelhardt (1991) and Lawless (1982). In this paper, we suggest an exact test based on conditional means and covariances of the censored sample using quadratic form.

2 Derivation of the test

Many times observations of failures are naturally occurring in order. In this case, it is convenient to terminate the experiment after observing the first r failures from n units. The principal advantage of such censoring called type-2 censoring is that it may take much less time for the first r failures of n items to occur, than for all itmes in a random sample of size r to fail. In this section we derive a test statistic based on a Type-2 censored sample and study its properties.

Let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(r)}, r \leq n$, be a Type-2 censored samples of a complete sample of size n from (1.1). Then the joint density of $X_{(1)}, X_{(2)}, \cdots, X_{(r)}$ is

$$f_{\underline{X}}(\underline{x};\theta,\beta) = \frac{n!}{(n-r)!}\left(\frac{\beta}{\theta}\right)^r \prod_{i=1}^{r} x_{(i)}^{\beta-1} e^{-\left[\sum_{i=1}^{r} x_{(i)}^{\beta-1}+(n-r)x_{(r)}^{\beta}\right]/\theta}, \quad (2.1)$$

$0 < X_{(1)} \le X_{(2)} \le \cdots \le X_{(r)} < \infty, \theta, \beta > 0$. Making the transformation $Y_{(i)} = X_{(i)}^{\beta_0}, \beta_0 > 0$, we consider the problem of testing $H_0 : \beta = \beta_0$ against $H_1 : \beta > \beta_0$ ($H_2 : \beta < \beta_0$), based on $(Y_{(1)}, Y_{(2)}, \cdots, Y_{(r)})$. The joint pdf of $Y_{(1)}, Y_{(2)}, \cdots, Y_{(r-1)}$ is then

$$f_{\underline{Y}}(\underline{y};\theta,\nu) = \frac{n!}{(n-r)!}\left(\frac{\nu}{\theta}\right)^r \prod_{i=1}^{r} y_{(i)}^{\nu-1} e^{-\left[\sum_{i=1}^{r} y_{(i)}^{\nu-1}+(n-r)y_{(r)}^{\nu}\right]/\theta}, \quad (2.2)$$

$0 < Y_{(1)} \le Y_{(2)} \le \cdots \le Y_{(r)} < \infty, \nu = \beta/\beta_0 > 0$. For given ν, $T = \sum_{i=1}^{r} Y_{(i)}^{\nu} + (n-r)Y_{(r)}^{\nu}$ is the complete sufficient statistic for θ, having the pdf

$$f_T(t;\theta) = \frac{1}{\theta^r(r-1)!} t^{r-1} e^{-t/\theta}, \ t > 0 \quad (2.3)$$

The joint pdf of $(Y_{(1)}, Y_{(2)}, \cdots, Y_{(r-1)})$ and T is

$$f(y_{(1)}, y_{(2)}, \cdots, y_{(r-1)}, t) = \frac{n!}{(n-r+1)!}\left(\frac{\nu}{\theta}\right)^{r-1}\left(\frac{1}{\theta}\right)\prod_{i=1}^{r-1} y_{(i)}^{\nu-1} e^{-t/\theta},$$

$0 < Y_{(1)}^{\nu} \le Y_{(2)}^{\nu} \le \cdots \le Y_{(r-1)}^{\nu} \le (t - \sum_{i=1}^{r-1} y_{(i)}^{\nu}/(n-r+1)) < \infty$. Therefore, the conditional pdf of $Y_{(1)}, Y_{(2)}, \cdots, Y_{(r-1)}$ given $T = t$ is

$$f(Y_{(1)}, Y_{(2)}, \cdots, Y_{(r-1)} | T = t) = \frac{n!(r-1)!}{(n-r+1)!}\left(\frac{\nu^{r-1}}{t}\right)\prod_{i=1}^{r-1} y_{(i)}^{\nu-1}, \quad (2.4)$$

$0 < Y_{(1)}^{\nu} \le Y_{(2)}^{\nu} \le \cdots \le Y_{(r-1)}^{\nu} \le (t - \sum_{i=1}^{r-1} y_{(i)}^{\nu}/(n-r+1))$.

The conditional distribution (2.4) does not depend on the nuisance parameter θ. Hence, we derive the test statistic for testing H_0 versus H_1 by treating the observations have come from (2.4). For this, we define

$$Q = (\underline{Y} - \underline{\mu_0})' {\sum}_0^{-1} (\underline{Y} - \underline{\mu_0}),$$

where $\underline{Y}' = (Y_{(1)}, Y_{(2)}, \cdots, Y_{(r-1)})$, $\underline{\mu_0}' = (\mu_1, \mu_2, \cdots, \mu_{r-1})$; $\mu_i = E_{H_0}[Y_{(i)} | T = t]$ and $\sum_0 = ((\delta_{ij}))$ is the conditional variance-covariance matrix of $\underline{Y}$ given $T = t$ computed under H_0. They can be obtained as follows:

We know that $Z_i' = (n-i+1)(Y_{(i)}^{\nu} - Y_{(i-1)}^{\nu})$, $i = 1, 2, \ldots, r$, $Y_{(0)} = 0, \nu$ known, are i.i.d exponential random variables with mean θ and $\sum_{i=1}^{r} Z_i = t$. Then the pdf of Z_i given $T = t$ is

$$f_{Z_i|T}(z_i|t) = \frac{(r-1)}{t}\left(1 - \frac{z_i}{t}\right)^{r-2}, \ 0 < z_i < t \tag{2.5}$$

According to the theorem 1.6.7 of Reiss (1989), the distribution of the vector $(Z_1/T, Z_2/T, \ldots, Z_r/T)'$ is the same as that of $(V_1, V_2, \ldots, V_r)'$ where V's are the spacings of a random sample of size r from the uniform distribution on (0,1). Further, the author has given the asymptotic distribution of this vector of spacings and other related results (see corollary 1.6.10 of Reiss (1989)).

Since $Y_{(i)}^{\nu} = \sum_{j=1}^{i} z_j/(n-j+1)$, under H_0

$$\begin{aligned}
\mu_i &= E_{H_0}(Y_{(i)}|t) \\
&= \frac{1}{(n-j+1)}\sum_{j=1}^{i} E(Z_j|T=t) \\
&= \frac{t}{r}\sum_{j=1}^{i}\frac{1}{(n-j+1)} \qquad (2.6)\\
\delta_{ii} &= V_{H_0}(Y_{(i)}|t) \\
&= V_{H_0}(\sum_{j=1}^{i}\frac{1}{(n-j+1)}Z_j|T=t) \\
&= \sum_{j=1}^{i}\frac{1}{(n-j+1)^2}V_{H_0}(Z_j|T=t) \\
&\quad + \sum_{j\neq k}^{i}\frac{1}{(n-j+1)(n-k+1)}Cov_{H_0}(Z_j, Z_k|T=t) \\
&= \sum_{j=1}^{i}\frac{1}{(n-j+1)^2}V_{H_0}(Z_j|T=t) \\
&\quad - \sum_{j\neq k}^{i}\frac{1}{(n-j+1)(n-k+1)(r-1)}V_{H_0}(Z_j|T=t) \\
&= \frac{r}{r-1}\sum_{j=1}^{i}\frac{1}{(n-j+1)^2}V_{H_0}(Z_j|T=t)
\end{aligned}$$

$$\begin{aligned} & -\sum_{j=1}^{i}\sum_{k=1}^{i}\frac{1}{(n-j+1)(n-k+1)(r-1)}V_{H_0}(Z_j|T=t) \\ = & \ \frac{r\nu}{r-1}\left[a_i-\frac{1}{r}b_{ii}\right], \end{aligned} \tag{2.7}$$

and

$$\begin{aligned} \delta_{ii} &= COV_{H_0}(Y_{(i)},Y_{(l)}|t), i\neq 1, i<1 \\ &= \sum_{j=1}^{i}\sum_{k=1}^{l}\frac{1}{(n-j+1)(n-k+1)}COV(Z_j,Z_k|T=t) \\ &= \sum_{j=1}^{i}\frac{1}{(n-j+1)^2}V_{H_0}(Z_j|T=t) \\ &\quad -\frac{1}{(r-1)}\sum_{j=1}^{i}\sum_{j\neq k=1}^{l}\frac{1}{(n-j+1)(n-k+1)}V_{H_0}(Z_j|T=t) \\ &= \frac{r}{(r-1)}\sum_{j=1}^{i}\frac{1}{(n-j+1)^2}V_{H_0}(Z_j|T=t) \\ &\quad -\frac{1}{(r-1)}\sum_{j=1}^{i}\sum_{k=1}^{l}\frac{1}{(n-j+1)(n-k+1)}V_{H_0}(Z_j|T=t) \\ &= \frac{r\nu}{(r-1)}\left[a_i-\frac{1}{r}b_{ii}\right], \end{aligned} \tag{2.8}$$

where

$$\begin{aligned} \nu &= V_{H_0}(Z_j|T=t)=\frac{(r-1)t^2}{r^2(r+1)}, \\ a_i &= \sum_{j=1}^{i}\frac{1}{(n-j+1)^2},\ \ i=1,2,\ldots,r-1 \end{aligned}$$

and

$$b_{il}=\sum_{j=1}^{i}\sum_{k=1}^{l}\frac{1}{(n-j+1)(n-k+1)},\ \ i=1,2,\ldots,r-1.$$

Using (2.7) and (2.8), the variance-covariance matrix $\sum_0$ of $(\underline{Y}|T=t)$ under H_0 is

$$\sum\nolimits_0 = k\left[A-\frac{1}{r}B\right], \tag{2.9}$$

where

$$k = \frac{r\nu}{r-1}$$

$$A = \begin{bmatrix} a_1 & a_1 & a_1 & \cdots & a_1 \\ a_1 & a_2 & a_2 & \cdots & a_2 \\ a_1 & a_2 & a_3 & \cdots & a_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_1 & a_1 & a_1 & \cdots & a_{r-1} \end{bmatrix},$$

and

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1(r-1)} \\ b_{12} & b_{22} & b_{23} & \cdots & b_{2(r-1)} \\ b_{13} & b_{23} & b_{33} & \cdots & b_{3(r-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{1(r-1)} & b_{2(r-1)} & b_{3(r-1)} & \cdots & b_{(r-1)(r-1)} \end{bmatrix},$$

Since A is non-singular matrix and B can be written as $B = \underline{CC}'$, where

$$\underline{C} = \left(\frac{1}{n} \; , \; \sum_{j=1}^{2} \frac{1}{(n-j+1)} \; , \; \cdots \; , \; \sum_{j=1}^{r-1} \frac{1}{(n-j+1)} \right),$$

using the result 2.8, pp.33 of Rao (1974), we have the inverse of $\sum_0$ as

$$\sum\nolimits_0^{-1} = \frac{1}{k}[A^{-1} + D];$$

where A^{-1} is given by

$$\begin{bmatrix} n^2 + (n-1)^2 & -(n-1)^2 & \cdots & 0 & 0 \\ -(n-1)^2 & (n-1)^2 + (n-2)^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & (n-r+3)^2 + (n-r+2)^2 & -(n-r+2)^2 \\ 0 & 0 & \cdots & -(n-r+2)^2 & (n-r+2)^2 \end{bmatrix},$$

and

$$D = \begin{bmatrix} 1 & 1 & \cdots & (n-r+2) \\ 1 & 1 & \cdots & (n-r+2) \\ \vdots & \vdots & \ddots & \vdots \\ (n-r+2) & (n-r+2) & \cdots & (n-r+2)^2 \end{bmatrix}.$$

Therefore, the expression for the test statistic based on the censored sample $Y_{(1)}, Y_{(2)}, \cdots, Y_{(r-1)}$ and the value t of T is

$$
\begin{aligned}
Q &= (\underline{Y}-\underline{\mu_0})'{\sum}_0^{-1}(\underline{Y}-\underline{\mu_0}) \\
&= \frac{1}{k}[(\underline{Y}-\underline{\mu_0})'A^{-1}(\underline{Y}-\underline{\mu_0}) + (\underline{Y}-\underline{\mu_0})'D(\underline{Y}-\underline{\mu_0})] \\
&= \frac{1}{k}[Q_1+Q_2], \text{ say.} \qquad (2.10)
\end{aligned}
$$

Now,

$$
\begin{aligned}
Q_1 &= (\underline{Y}-\underline{\mu_0})'A^{-1}(\underline{Y}-\underline{\mu_0}) \\
&= \sum_{j=1}^{r-2}[(n-j+1)^2+(n-j)^2](Y_{(j)}-\mu_j)^2 \\
&\quad -2\sum_{j=1}^{r-2}(n-j)^2(Y_{(j)}-\mu_j)(Y_{(j+1)}-\mu_{j+1}) \\
&\quad +(n-r+2)^2(Y_{(r-1)}-\mu_{r-1})^2 \\
&= \sum_{j=1}^{r-1}(n-j+1)^2(Y_{(j)}-\mu_j)^2 \\
&\quad +\sum_{j=1}^{r-2}(n-j)^2[(Y_{(j)}-\mu_j)-(Y_{(j+1)}-\mu_{j+1})]^2 \\
&\quad -\sum_{j=1}^{r-2}(n-j)^2(Y_{(j+1)}-\mu_{j+1})^2 \\
&= n^2(Y_{(1)}-\mu_{(1)})^2+\sum_{j=1}^{r-2}(n-j)^2[(Y_{(j)}-Y_{(j+1)})-(\mu_j-\mu_{j+1})]^2 \\
&= \sum_{j=1}^{r-2}\left[(n-j)(Y_{(j+1)}-Y_{(j)})-\frac{t}{r}\right]^2 \\
&= \sum_{i=1}^{r-1}\left[(n-i+1)(Y_{(i)}-Y_{(i-1)})-\frac{t}{r}\right]^2. \qquad (2.11)
\end{aligned}
$$

and

$$
\begin{aligned}
Q_2 &= (\underline{Y}-\underline{\mu_0})'D(\underline{Y}-\underline{\mu_0}) \\
&= \sum_{i=1}^{r-2}(Y_{(i)}-\mu_i)+(n-r+2)(Y_{(r-1)}-\mu_{r-1})^2
\end{aligned}
$$

$$= \left[(n-r+1)(Y_{(r)} - Y_{(r-1)}) - \frac{t}{r}\right]^2. \qquad (2.12)$$

Hence, using (2.11) and (2.12) in (2.10) we have finally,

$$Q = \frac{r(r+1)^2}{t^2} \sum_{i=1}^{r} \left[(n-i+1)(Y_{(i)} - Y_{(i-1)}) - \frac{t}{r}\right]^2. \qquad (2.13)$$

This statistic is again based on the given condition $T = t$. So, we modify this statistic by replacing t by its corresponding random variable T and propose the new statistic based on a type-2 censored sample $X_{(1)}, X_{(2)}, \cdots, X_{(r)}$ as

$$Q^* = r(r+1)\left[\sum_{i=1}^{r} \frac{(n-i+1)^2}{T^2}(X_{(i)}^{\beta_0} - X_{(i-1)}^{\beta_0})^2 - \frac{1}{r}\right], \qquad (2.14)$$

where $T = \sum_{i=1}^{r} X_{(i)}^{\beta_0} + (n-r)X_{(r)}^{\beta_0}$.

To find the exact mean and variance of Q^*, under H_0, we again consider $Z_i = (n-i+1)(X_{(i)}^{\beta_0} - X_{(i-1)}^{\beta_0})$, $i = 1, 2, \ldots, r$, $X_{(0)} = 0$. Then we have $Q^* = r(r+1)[V - 1/r]$, where $V = \sum_{i=1}^{r} Z_i^2/T^2$ and $T = \sum_{i=1}^{r} Z_i$. Since $E_{H_0}(V) = \frac{2}{r+1}$ and $E_{H_0}(V^2) = \frac{4(r+5)}{(r+1)^2(r+2)(r+3)}$, we have

$$E_{H_0}(Q^*) = r - 1$$

and

$$V_{H_0}(Q^*) = \frac{4r^2(r-1)}{(r+2)(r+3)}.$$

Hence, the standardized form of Q^* is

$$\begin{aligned} Q^*_{(sd)} &= \frac{Q^* - E_{H_0}(Q^*)}{\sqrt{Var(Q^*)}} \\ &= \sqrt{\frac{(r+2)(r+3)}{(r-1)}} \\ &\quad \left[\frac{(r+1)\sum_{i=1}^{r}(n-i+1)^2(X_{(i)}^{\beta_0} - X_{(i-1)}^{\beta_0})^2}{2\left(\sum_{i=1}^{r} X_{(i)}^{\beta_0} + (n-r)X_{(r)}^{\beta_0}\right)^2} - 1\right]. \end{aligned}$$

Since we could not find the expression for $E_{H_1}(Q^*_{(sd)})$, we observed the direction of the test procedure by simulating its values for different $n = 10, 20, 30$ with censoring proportion $p, 0 < p < 1$ and for different values of β and found that $E_{H_1}(Q^*_{(sd)}) > E_{H_0}(Q^*_{(sd)})$ for $\beta \notin [\beta_0, g(n)]$, where $g(n)$ is a function of n which tends to β_0 as $n \to \infty$. Hence, the test procedure is to reject H_0 for large values of $Q^*_{(sd)}$ when $\beta < \beta_0$ and $\beta > g(n)$.

3 Asymptotic null distribution of Q^*

Define $V_i = (Z_i, Z_i^2)'$, $i = 1, 2, \ldots, r$, where

$$Z_i = (n - i + 1)(X^{\beta_0}_{(i)} - X^{\beta_0}_{(i-1)}).$$

It is noted that $\{V_i,\ i = 1, 2, \ldots, r\}$ are iid random vectors in view of the iid nature of $\{X_i^{\beta_0},\ i = 1, 2, \ldots, r\}$. Applying Multivariate CLT to $\{V_i,\ i = 1, 2, \ldots, r\}$, the asymptotic distribution of $\sqrt{r}\ (\underline{V} - E(\underline{V}))$ is bivariate normal with zero mean and dispersion matrix $D(\underline{V})$, where $\underline{V} = (V_1, V_2)'$, $V_1 = \sum_{i=1}^r Z_i/r$ and $V_2 = \sum_{i=1}^r Z_i^2/r$. Also we have $E(\underline{V}) = (\theta, 2\theta^2)'$ and $D(\underline{V}) = \begin{bmatrix} \theta^2 & 4\theta^3 \\ 4\theta^3 & 20\theta^4 \end{bmatrix}$. Then the asymptotic distribution of $g(V_1, V_2) = V_2/V_1^2$ is normal with mean 2 and variance $4/r$. Hence

$$Q^* = (r + 1)[V_2/V_1^2 - 1] \sim N(r + 1, 4(r + 1)^2/r).$$

4 A power study of the test

First obtain the percentage points of the distribution of $Q^*_{(sd)}$ by Monte Carlo method. For this, we generate 5000 random samples of different size n from Weibull distribution with $\theta = \beta = 1$ and then construct type-2 censored samples with censoring proportion $p = r/n$ $(0 < p < 1)$. Using these samples we simulate the percentage points of the distribution of $Q^*_{(sd)}$. The results are presented in Table 1 for $p = 0.1$.

Table 1. The percentage points of the distribution of $Q^*_{(sd)}$ for $p = 0.1$.

n	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.95$	$\alpha = 0.99$
10	-1.39	-1.16	0.95	2.07
20	-1.50	-1.22	0.44	1.23
30	-1.55	-1.22	0.14	0.82
40	-1.58	-1.26	-0.05	0.67
50	-1.65	-1.31	-0.24	0.53
60	-1.70	-1.33	-0.40	0.44
70	-1.72	-1.34	-0.51	0.43
80	-1.73	-1.34	-0.62	0.34
90	-1.75	-1.35	-0.74	0.42
100	-1.76	-1.36	-0.79	0.38
120	-1.81	-1.39	-0.85	0.37

Using the above percentage points we compute the power of the test statistic $Q^*_{(sd)}$ for Weibull alternative through simulation for different values of n and β/β_0. The powers are compared with the test proposed by Bain and Engelhardt (1986) test and are given in tables 2 and 3 for different censoring schemes.

Table 2. Power of $Q^*_{(sd)}$ test for values of $\beta < \beta_0$.

n	$\beta/\beta_0 = 0.2$		$\beta/\beta_0 = 0.4$		$\beta/\beta_0 = 0.6$		$\beta/\beta_0 = 0.8$	
	1%	5%	1%	5%	1%	5%	1%	5%
10	.89	.96	.50	.71	.21	.41	.08	.22
	.94	.99	.82	.92	.33	.52	.05	.21
20	1.00	1.00	.84	.94	.45	.66	.18	.36
	1.00	1.00	.98	.99	.54	.69	.12	.31
30	1.00	1.00	.95	.99	.63	.81	.30	.52
	1.00	1.00	.99	1.00	.74	.91	.19	.43
50	1.00	1.00	1.00	1.00	.81	.95	.40	.69
	1.00	1.00	1.00	1.00	.86	.97	.50	.59
70	1.00	1.00	1.00	1.00	.88	.98	.76	.82
	1.00	1.00	1.00	1.00	.94	.99	.46	.74
90	1.00	1.00	1.00	1.00	.99	1.00	.51	.89
	1.00	1.00	1.00	1.00	.99	1.00	.48	.82
120	1.00	1.00	1.00	1.00	.97	1.00	.55	.94
	1.00	1.00	1.00	1.00	.99	1.00	.56	.88

(The second entries are correspond to Bain and Engelhardt (1986) test).

Table 3. Power of $Q^*_{(sd)}$ test for values of $\beta > \beta_0$.

n	$\beta/\beta_0 = 1.2$		$\beta/\beta_0 = 1.4$		$\beta/\beta_0 = 1.8$		$\beta/\beta_0 = 2.0$	
	1%	5%	1%	5%	1%	5%	1%	5%
10	.04	.13	.05	.17	.13	.30	.17	.38
	.02	.10	.05	.22	.11	.28	.16	.36
20	.11	.26	.18	.36	.41	.64	.56	.76
	.04	.18	.14	.27	.44	.52	.49	.82
30	.19	.41	.32	.54	.66	.84	.81	.93
	.11	.32	.32	.54	.55	.83	.75	.91
50	.30	.60	.49	.76	.89	.98	.96	.99
	.17	.45	.38	.65	.82	.96	.98	.99
70	.35	.75	.60	.88	.96	1.00	1.00	1.00
	.20	.47	.53	.85	.97	1.00	1.00	1.00
90	.40	.85	.68	.95	.98	1.00	1.00	1.00
	.35	.57	.65	.97	.99	1.00	1.00	1.00
120	.44	.89	.77	.98	.99	1.00	1.00	1.00
	.39	.68	.72	.99	.99	1.00	1.00	1.00

(The second entries are correspond to Bain and Engelhardt (1986) test).

5 Example

We now present an example from Meeker and Escobar (1998). For readability, we reproduce below, the data on the distance to failure for 38 Vehicle Shock Absorbers:

6700, 6950, 7820, 8790, 9120, 9660, 9820, 11310,
11690, 11850, 11880, 12140, 12200, 12870, 13150, 13330,
13470, 14040, 14300, 17520, 17540, 17890, 18450, 18960,
18980, 19410, 20100, 20100, 20150, 20320, 20900, 22700,
23490, 26510, 27410, 27490, 27890, 28100.

Meeker and Escobar (1998), have obtained maximum likelihood estimates and confidence intervals for the parameters under Weibull and lognormal distributions respectively and concluded Weibull as the best model to represent the above data. Suppose we would like to test $H_0 : \beta = 1$ against $H_1 : \beta > 1$. For this, the computed value of $Q^*_{(sd)}$ is 2.0477 and the corresponding percentile points for

1% and 5% level of significance are -1.57 and -1.25 respectively. Since the calculated value of $Q^*_{(sd)}$ is larger than the tabulated values, we conclude that Weibull distribution is an accepted model for the data. The estimated power against the alternatives $H_1 : \beta > 1$ is close to 1.

6 Conclusions

We have proposed a test Q^* for testing $H_0 : \beta = \beta_0$ against $H_1 : \beta > \beta_0$ ($H_2 : \beta < \beta_0$) and through power study shown that the test performs very well for $\beta < \beta_0$ even for small samples, but for $\beta > \beta_0$ it perform well only for large samples. Thus, we recommend the test Q^* for identifying Weibull DFR alternatives.

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