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Improved Estimation in Rayleigh Type-II Censored Data under a Bounded Loss Utilizing a Point Guess Value

Mehran Naghizadeh Qomi

Department of Statistics, University of Mazandaran, Babolsar, Iran
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Abstract. The problem of shrinkage testimation (test-estimation) for the Rayleigh scale parameter θ based on censored samples under the reflected gamma loss function is considered. We obtain a minimum risk estimator in a subclass and compute its risk. A shrinkage testimator based on the acceptance or rejection of the null hypothesis $H_0 : \theta = \theta_0$ is constructed, where θ_0 is a point guess value of θ. The risk of the proposed shrinkage testimator is computed numerically and compared with the minimum risk estimator. A data set is analyzed for illustrative purposes.

Keywords. Censored data, Rayleigh distribution, Reflected gamma loss function.

MSC: 62F03; 62F10.

1 Introduction

In classical methods of statistics, the parameter of interest is estimated based on a random sample using natural estimators such as maximum likelihood estimator (MLE) provided that it exists. In a Bayesian perspective, a Bayes estimator is derived by employing a flexible prior distribution for the parameter of interest. In some situations, the experimenter has some prior information about the parameter in the form of a point guess value. For example, a producer may know that the mean of life times of electronic

Mehran Naghizadeh Qomi (m.naghizadeh@umz.ac.ir)

components in an accelerated life test is close to 600 minutes. Therefore, he/she can improve the natural estimator by shrinking it towards the guess value and construct a linear shrinkage estimator in the hope that it will perform better than the natural estimator.

When a point guess value θ_0 of the parameter θ is available, Thompson (1968) proposed the shrinkage estimator

$$\hat{\theta}_s = k\hat{\theta} + (1-k)\theta_0, \quad 0 \leq k \leq 1, \tag{1.1}$$

where k is the shrinkage factor. The value of k near to zero (one) implies strong belief in the guess value θ_0 (sample values). Information in the guess value (non-sample information) can be expressed in the form of a preliminary test $H_0: \ \theta = \theta_0$ against $H_0: \ \theta \neq \theta_0$. Based on acceptance or rejection of the null hypothesis, we can construct some shrinkage testimators (test-estimators) which have smaller risk than the natural estimators in some interval around the guess value, see Ahmed (1992).

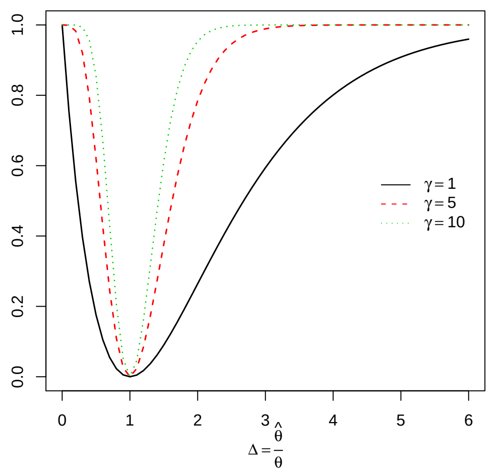
The problem of shrinkage testimation has received significant attention in recent years. A few of the many works that have recently appeared in this area are as follow: Singh *et al*. (2007) considered the shrinkage testimation for the Pareto scale-parameter under the linear-exponential (LINEX) loss function. Prakash and Pandey (2007) and Prakash and Singh (2008) provided some shrinkage testimators for the variance of a normal distribution and the scale-parameter of the exponential distribution under the LINEX loss, respectively. New works in this area are performed by Mirfarah and Ahmadi (2014), Belaghi *et al*. (2015), Naghizadeh Qomi and Barmoodeh (2015) and Kiapour and Naghizadeh (2016).

For estimating a scale-parameter, the common loss is the scale invariant squared error loss (SISEL) which is symmetric. Some asymmetric loss functions such as entropy and LINEX loss are motivated in response to the criticism of the SISEL. However, these loss functions with their infinite maximum value are not appropriate in practice. In some estimation problems, the use of unbounded loss function may be inappropriate. For example, in estimating the mean life θ of the components of an aircraft, the amount of loss for estimating θ by an estimator is essentially bounded. In the present paper, we deal with the shrinkage testimation of a Rayleigh scale-parameter under Reflected Gamma Loss (RGL) function. The RGL function is a simple transformation of the gamma density and introduced by Spiring and Yeung (1998) and is defined by

$$L(\Delta) = k^{\star}\left\{1 - \Delta^{\gamma} e^{-\gamma(\Delta-1)}\right\}, \quad \Delta = \frac{\hat{\theta}}{\theta}, \tag{1.2}$$

where $k^{\star} > 0$ is the maximum loss, $\gamma > 0$ is a shape parameter and $\hat{\theta}$ is an estimator of θ. This loss can be used when $\Omega = R^+$ where Ω is the parameter space. The

Figure 1: Plot of the RGL function for $k^{\star} = 1$ and selected values of $\gamma = 1, 5, 10$.



RGL function is a bounded and asymmetric function of Δ, but not convex in Δ, and is essentially a gamma density function flipped upside down, whence its name, see Figure 1. Towhidi and Behboodian (1999,2002), Meghnatisi and Nematollahi (2009), Kaminskaa and Porosinskia (2009) and Naghizadeh Qomi *et al.* (2015) have used the RGL function (1.2) in some estimation problems. Clearly the value of $k^{\star} > 0$ does not have any influence on our results, therefore, without loss of generality, we shall take $k^{\star} = 1$ in the rest of the paper. The paper is organized as follows. we first present the considered model and data. Then, we derive the MLE ($\hat{\theta}$) and the minimum risk estimator within the class $c\hat{\theta}$ under the RGL function. We show that the MLE (the estimator corresponding to $c = 1$) is the minimum risk estimator. In Section 3, we propose a shrinkage testimator and compare it with the minimum risk estimator. Moreover, we compare the performance of the three proposed shrinkage testimators. A real data set is used for illustrative purposes in Section 4. We end up with a concluding remark in Section 5.

2 Model and Data

Let X have a one-parameter Rayleigh distribution with probability density function (p.d.f.)

$$f(x;\theta) = \frac{x}{\theta} \exp\left\{-\frac{x^2}{2\theta}\right\}, \quad x > 0, \tag{2.1}$$

and the corresponding comulative density function (c.d.f.) as

$$F(x;\theta) = 1 - \exp\left\{-\frac{x^2}{2\theta}\right\}, \quad x > 0. \tag{2.2}$$

We will use $X \sim Ray(\theta)$ to mean that the random variable X is distributed according to the Rayleigh distribution with parameter θ.

The linear and increasing failure rate of the Rayleigh distribution is

$$r(t;\theta) = \frac{f(t;\theta)}{1 - F(t;\theta)} = \frac{t}{\theta}, \tag{2.3}$$

which makes it an appropriate distribution for modeling lifetimes of components. Several types of electro-vacuum devices have this feature, see Polovko (1968).

Assume that n randomly selected devices are placed on a life test simultaneously and the test will be finished immediately after r components have failed. Here, r is a specified integer between 1 and n and is chosen before the data are collected. Therefore, r is fixed and the length of the experiment is a random variable. Let $X_{1:n}, \cdots, X_{r:n}$ denote the type-II right censored observations from the Rayleigh model given in (2.1). Then, we have the following well-known results due to Arnold *et al*. (2008):

(i) The joint p.d.f. of $\mathbf{X} = (X_{1:n}, \cdots, X_{r:n})$ is given by

$$f_{X_{1:n},\cdots,X_{r:n}}(x_1, \cdots, x_r) = \frac{n! \prod_{i=1}^r x_{i:n}}{(n-r)!\theta^r} \exp\left\{-\frac{\sum_{i=1}^r x_{i:n}^2 + (n-r)x_{r:n}^2}{2\theta}\right\}. \tag{2.4}$$

(ii) The MLE of θ, denoted by $\hat{\theta}$, is

$$\hat{\theta} = \frac{\sum_{i=1}^r X_{i:n}^2 + (n-r)X_{r:n}^2}{2r}. \tag{2.5}$$

(iii) The spacings $Z_i = (n-i+1)(X^2_{i:n} - X^2_{i-1:n})/\theta$ for $i = 1, ..., r$ $(X_{0:n} \equiv 0)$ constitute a random sample of χ^2_2, a chi-square distribution with 2 degrees of freedom, and then $2r\hat{\theta}/\theta \sim \chi^2_{2r}$.

Remark 1. Since the RGL function is bounded, by a result in Basu (1955), the uniformly minimum risk unbiased estimator of any unknown parameter does not exist under the RGL function.

In the following Lemma, we derive the minimum risk estimator within the class $c\hat{\theta}$ under RGL.

Lemma 2.1. *Let $X_{1:n}, \cdots, X_{r:n}$ denote the type-II right censored observations from the Rayleigh model. Then, $\hat{\theta}$ is the minimum risk estimator within the class $c\hat{\theta}$ under RGL with the risk*

$$R(\theta, \hat{\theta}) = 1 - \frac{r^r e^\gamma \Gamma(r+\gamma)}{\Gamma(r)(r+\gamma)^{r+\gamma}}. \tag{2.6}$$

Proof. The risk of $c\hat{\theta}$ under the RGL function is

$$\begin{aligned} R(\theta, c\hat{\theta}) &= 1 - E\left[\left(\frac{c\hat{\theta}}{\theta}\right)^\gamma e^{-\gamma(\frac{c\hat{\theta}}{\theta}-1)}\right] \\ &= 1 - \left(\frac{ce}{r}\right)^\gamma E\left[W^\gamma e^{-\frac{c\gamma W}{r}}\right] = 1 - \left(\frac{ce}{r}\right)^\gamma \int_0^\infty w^\gamma e^{-\frac{c\gamma w}{r}} \frac{w^{r-1}e^{-w}}{\Gamma(r)} dw \\ &= 1 - \frac{c^\gamma}{(r+c\gamma)^{r+\gamma}} A(r,\gamma), \end{aligned} \tag{2.7}$$

where $A(r,\gamma) = r^r e^\gamma \Gamma(r+\gamma)/\Gamma(r)$ and $W = \frac{r\hat{\theta}}{\theta}$ has a *Gamma*$(r, 1)$ distribution. The first derivative of $R(\theta, c\hat{\theta})$ with respect to c is given by

$$\frac{\partial R(\theta, c\hat{\theta})}{\partial c} = A(r,\gamma) \frac{r\gamma(c-1)c^{\gamma-1}}{(r+c\gamma)^{r+\gamma+1}}.$$

Then, the risk function has a unique minimum at $c = 1$ and is strictly decreasing for $c < 1$ and strictly increasing for $c > 1$. There are unique points $c_1 < 1 < c_2$ such that $R(\theta, c\hat{\theta})$ is a convex function in (c_1, c_2) and concave in $(-\infty, c_1)$ and (c_2, ∞). Then, the minimum risk estimator within the class $c\hat{\theta}$ is $\hat{\theta}$, which is the MLE of θ. □

3 Shrinkage Testimators

When a point guess value θ_0 of θ is available, a preliminary test $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta \neq \theta_0$ may be performed to check that θ_0 is in the vicinity of θ or not. A Likelihood Ratio Test (LRT) statistic is $U = 2r\hat{\theta}/\theta \sim \chi^2_{2r}$ that has a rejection region of the form $2r\hat{\theta}/\theta_0 > q_2$ or $2r\hat{\theta}/\theta_0 < q_1$, where q_1 and q_2 are the values of the lower and upper $100\frac{\alpha}{2}\%$ points of the chi-square distribution with $2r$ degrees of freedom, i.e. $P_{\theta_0}(U < q_1) = P_{\theta_0}(U > q_2) = \alpha/2$.

3.1 Form of Shrinkage Testimator and It's Risk

We can construct a shrinkage testimator based on the acceptance or rejection of H_0. The proposed shrinkage testimator, say $\hat{\theta}_{st}$, is $k\hat{\theta} + (1-k)\theta_0$, if H_0 is accepted, or $\hat{\theta}$, otherwise. If H_0 is accepted at the significance level α, then we have

$$\Pr\left(q_1 \leq \frac{2r\hat{\theta}}{\theta_0} \leq q_2\right) = 1 - \alpha.$$

Therefore, the proposed shrinkage testimator can be written as

$$\hat{\theta}_{st} = \begin{cases} k\hat{\theta} + (1-k)\theta_0 & r_1 \leq Y_r \leq r_2 \\ \hat{\theta} & Y_r < r_1 \ or \ Y_r > r_2, \end{cases} \tag{3.1}$$

where $Y_r = r\hat{\theta}$, $r_1 = q_1\theta_0/2$ and $r_2 = q_2\theta_0/2$. The risk function of the shrinkage testimator $\hat{\theta}_{st}$ under the RGL is computed as

$$\begin{aligned} R(\theta, \hat{\theta}_{st}) &= 1 - E\left[\left(\frac{\hat{\theta}_{st}}{\theta}\right)^{\gamma} e^{-\gamma(\frac{\hat{\theta}_{st}}{\theta}-1)}\right] \\ &= 1 - E\left\{\left(\frac{k\hat{\theta} + (1-k)\theta_0}{\theta}\right)^{\gamma} e^{-\gamma(\frac{k\hat{\theta}+(1-k)\theta_0}{\theta}-1)} I(B)\right\} \\ &- E\left\{\left(\frac{\hat{\theta}}{\theta}\right)^{\gamma} e^{-\gamma(\frac{\hat{\theta}}{\theta}-1)} I(B^c)\right\}, \end{aligned}$$

where $B = \{Y_r : \ r_1 \leq Y_r \leq r_2\}$, B^c is the complement of B, $I(B)$ and $I(B^c) = 1 - I(B)$ denote the indicator functions of B and B^c, respectively. Using $W = \frac{Y_r}{\theta} \sim Gamma(r, 1)$ and, after some simple computations, we get

$$R(\theta, \hat{\theta}_{st}) = \int_{w_1}^{w_2} \left(\frac{w}{r}\right)^{\gamma} e^{-\gamma(\frac{w}{r}-1)} g(w)dw - \frac{r^r e^{\gamma}\Gamma(r+\gamma)}{\Gamma(r)(r+\gamma)^{r+\gamma}}$$

$$- \int_{w_1}^{w_2} \left[\frac{kw}{r} + (1-k)\delta_0 \right]^{\gamma} e^{-\gamma(\frac{kw}{r}+(1-k)\delta_0-1)} g(w)dw + 1, \tag{3.2}$$

where $\delta_0 = \frac{\theta_0}{\theta}$, $w_1 = q_1\delta_0/2$, $w_2 = q_2\delta_0/2$ and $g(w)$ is the density of W. The risk function of $\hat{\theta}_{st}$ given in (3.2) can be computed numerically using the statistical package `R` version `3.1.2`.

3.2 Performance of Shrinkage Testimators

For comparison purposes, the relative efficiency (R.E.) between the shrinkage testimator $\hat{\theta}_{st}$ and $\hat{\theta}$ is defined as

$$RE(\hat{\theta}_{st}, \hat{\theta}) = \frac{R(\theta, \hat{\theta})}{R(\theta, \hat{\theta}_{st})}, \tag{3.3}$$

and have been plotted in Figures 2-5 for various values of r, α, k, γ and δ_0.

- Figures 2-3 show the plots of the R.E. between $\hat{\theta}_{st}$ and $\hat{\theta}$ for selected values of $\alpha = 0.01, 0.05, \gamma = 1, 5, r = 5, 10, 20$ and $k = 0.2(0.2)0.8$ with respect to $\delta_0 = \theta_0/\theta$. A horizontal line in 1 has been inserted for a better comparison. From these figures, we observe that the shrinkage testimators perform better than $\hat{\theta}$, when δ_0 is close to 1. Also, for fixed r, α and γ when δ_0 is near to 1, the testimators with small k are more efficient than other testimators.

- The values of R.E. between $\hat{\theta}_{st}$ and $\hat{\theta}$ for selected values of $\alpha = 0.01, 0.05, \gamma = 1, 5$, $r = 5(5)20$ and $\delta_0 = 1$ with respect to k have been plotted in Figure 4. This figure shows that the R.E. is decreasing in k and the shrinkage testimators with small k and r work well for fixed α and γ.

- Keeping $\gamma = 1$, the values of R.E. between $\hat{\theta}_{st}$ and $\hat{\theta}$ are plotted in Figure 5 for selected values of $r = 5, 10, 20$ and $\alpha = 0.01, 0.05, 0.1$ with respect to k when $\delta_0 = 1$. We see from Figure 5 that the testimators constructed with lower values of α, i.e. $\alpha = 0.01$, are better than other testimators for fixed r and γ.

For a comparison between shrinkage testimators with respect to γ, we plotted the risks of these testimators in Figure 6. Keeping $\alpha = 0.01$, we see that a testimator with lower γ, i.e. $\gamma = 1$, has small risks for fixed r.

Figure 2: Plots of R.E. between $\hat{\theta}_{st}$ and $\hat{\theta}$ for $\alpha = 0.01$, $\gamma = 1$ and selected values of $r = 5, 10, 20$ and $k = 0.2(0.2)0.8$ with respect to δ_0.

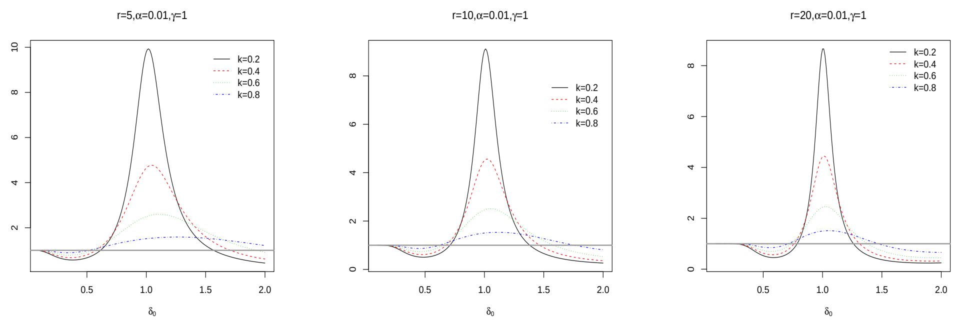


Figure 3: Plots of R.E. between $\hat{\theta}_{st}$ and $\hat{\theta}$ for $\alpha = 0.05$, $\gamma = 5$ and selected values of $r = 5, 10, 20$ and $k = 0.2(0.2)0.8$ with respect to δ_0.

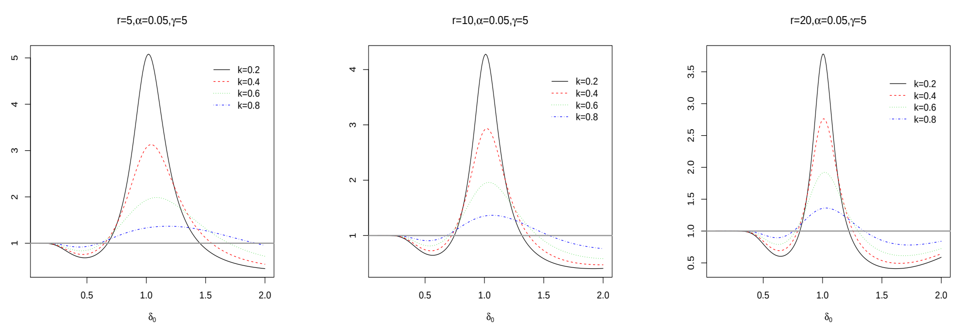


Figure 4: Plots of R.E. between $\hat{\theta}_{st}$ and $\hat{\theta}$ for selected values of $\alpha = 0.01, 0.05$, $\gamma = 1, 5$, $r = 5(5)20$ and $\delta_0 = 1$ with respect to k.

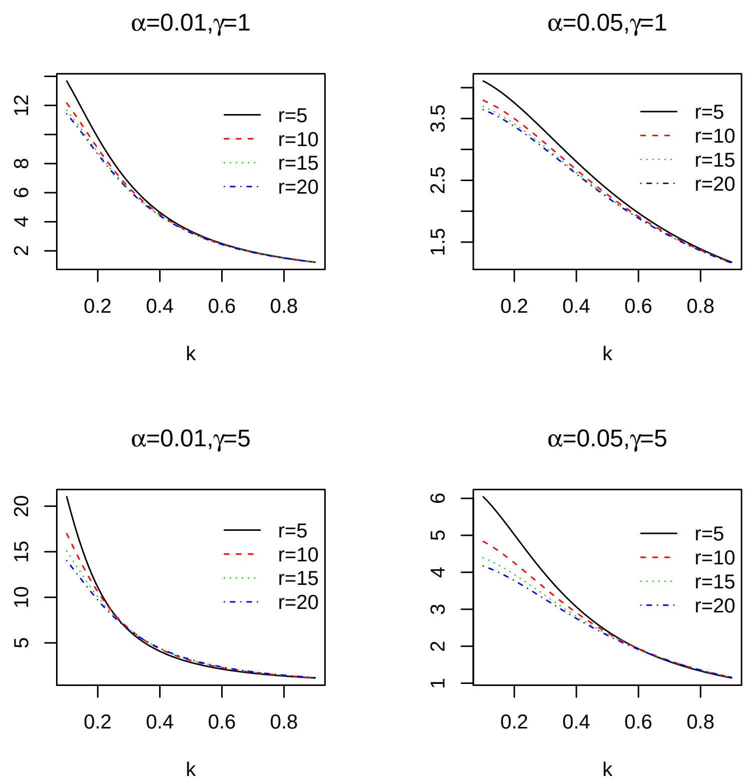


Figure 5: Plots of R.E. between $\hat{\theta}_{st}$ and $\hat{\theta}$ for $\gamma = 1$, $\delta_0 = 1$ and selected values of $r = 5, 15, 20$ and $\alpha = 0.01, 0.05, 0.1$ with respect to k.

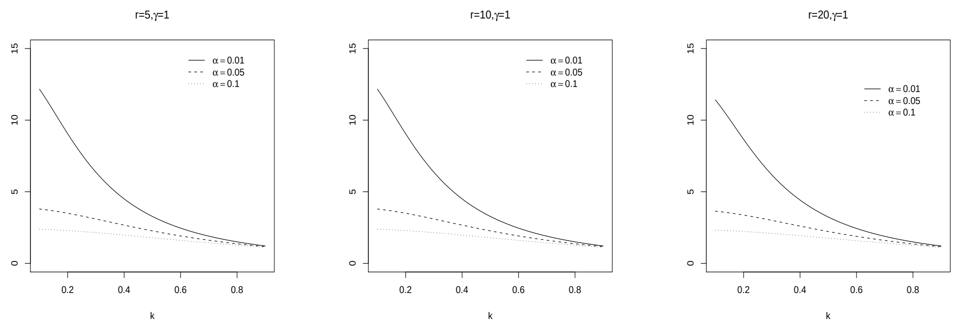


Figure 6: Plots for the values of risks of $\hat{\theta}_{st}$ for $\alpha = 0.01$, $\delta_0 = 1$ and selected values of $r = 5, 15, 20$ and $\gamma = 1, 5, 10$ with respect to k.

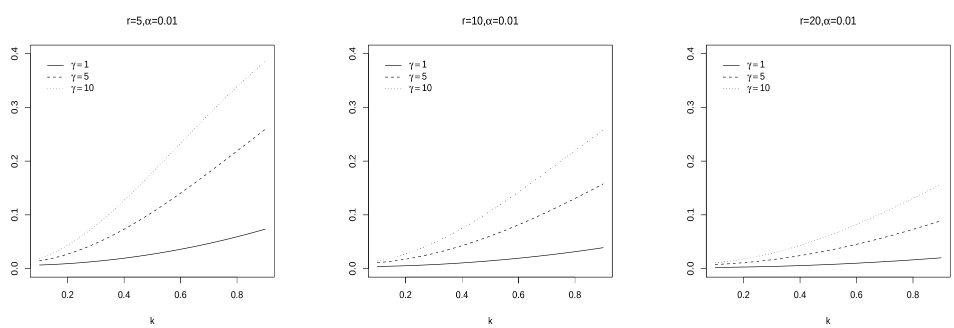
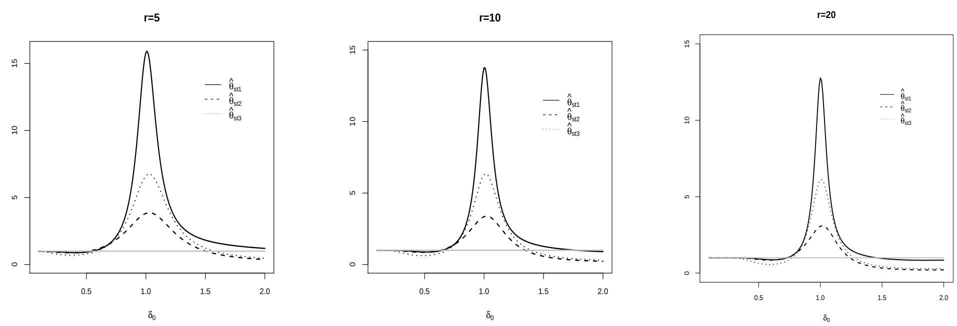


Figure 7: Plots of R.E. between $\hat{\theta}_{sti},\ i = 1, 2, 3$ and $\hat{\theta}$ for $\alpha = 0.01$, $\gamma = 1$ and selected values of $r = 5, 15, 20$ with respect to δ_0.



3.3 Selection of k

A choice of shrinkage factor k is to choose the parameter k in a data-driven fashion by explicitly minimizing the risk of the shrinkage estimator $\hat{\theta}_s$ given in (1.1). The risk of the shrinkage estimator $\hat{\theta}_s$ under the RGL function is given by

$$\begin{aligned} R(\theta, \hat{\theta}_s) &= 1 - E\Big[\Big(\frac{\hat{\theta}_s}{\theta}\Big)^{\gamma} e^{-\gamma(\frac{\hat{\theta}_s}{\theta}-1)}\Big] \\ &= 1 - E\Big[\Big(\frac{k\hat{\theta} + (1-k)\theta_0}{\theta}\Big)^{\gamma} e^{-\gamma(\frac{k\hat{\theta}+(1-k)\theta_0}{\theta}-1)}\Big] \\ &= 1 - \int_0^{\infty} \Big[\frac{kw}{r} + (1-k)\delta_0\Big]^{\gamma} e^{-\gamma(\frac{kw}{r}+(1-k)\delta_0-1)} g(w)dw, \end{aligned} \tag{3.4}$$

where δ_0 and $g(w)$ were defined in the previous subsection. The minimizing value of $k \in [0, 1]$ is given by

$$k_1 = \begin{cases} 0 & k \leq 0 \\ k & 0 \leq k \leq 1 \\ 1 & k \geq 1, \end{cases} \tag{3.5}$$

which is obtained numerically. We call the shrinkage testimator $\hat{\theta}_{st1}$ for the corresponding shrinkage factor k_1.

When the null $H_0 : \theta = \theta_0$ is accepted, we can select two shrinkage factors. The first is based on Waikar *et al.* (1984). The inequality $q_1 \leq 2Y_r/\theta_0 \leq q_2$ implies that

$$0 \leq k_2 = \frac{1}{q_2 - q_1}\Big(\frac{2Y_r}{\theta_0} - q_1\Big) \leq 1.$$

Then, the shrinkage factor k_2 constructs the shrinkage testimator $\hat{\theta}_{st2}$.

The second is based on Prakash and Singh (2008). Under H_0, we have $E(Y_r) = r\theta_0$ and, from the inequality $q_1 \leq 2Y_r/\theta_0 \leq q_2$, we get $q_1 \leq 2r \leq q_2$ which implies that $q_1/(2r) \leq 1$. If we want small values of the shrinkage factor, we can take $q_1/(2r) \approx 1$. Hence,

$$\frac{2r}{q_2 - q_1}\left(\frac{Y_r/\theta_0}{r} - \frac{q_1}{2r}\right) \approx \frac{2r}{q_2 - q_1}\left(\frac{Y_r}{r\theta_0} - 1\right).$$

Therefore, the shrinkage factor k_3 for constructing the shrinkage testimator $\hat{\theta}_{st3}$ is given by

$$k_3 = \frac{2n}{q_2 - q_1}\left|\frac{Y_n}{n\theta_0} - 1\right|,$$

where the absolute value is for avoiding from negative values.

Figure 7 presents the R.E. between the shrinkage testimators $\hat{\theta}_{sti}$, $i = 1, 2, 3$ and $\hat{\theta}$ for selected values of $r = 5, 10, 20$, $\alpha = 0.01$ and $\gamma = 1$ with respect to δ_0 (more figures are provided, but not presented here). It is observed that all testimators perform better than $\hat{\theta}$ for δ_0 closer to 1. Moreover, the testimator $\hat{\theta}_{st1}$ is more efficient than other testimators for the values of δ_0 near to one.

4 A real data set

The following data set are the failure times (in minutes) for a sample of fifteen electronic components in an accelerated life test:

$$1.4, 5.1, 6.3, 10.8, 12.1, 18.5, 19.7, 22.2, 23, 30.6, 37.3, 46.3, 53.9, 59.8, 66.2.$$

These data are from Lawless (2003). Mirmostafaee *et al.* (2016) checked the adequacy of the fitness of the Rayleigh distribution with $\theta = 580.5973$ using the Kolmogorov-Smirnov (K-S) test with the test statistic $D = 0.2341$ and a corresponding $p - value = 0.3837$. Hence, we cannot claim that the Rayleigh distribution is an inadequate distribution for modeling these data.

Assume that we have failed to observe the last seven ordered data so that $r = 8$ and $n = 15$. The MLE of θ is $\hat{\theta} = 312.7356$. If we consider $\gamma = 1, 5$, then the risk of $\hat{\theta}$ is 0.0581 and 0.2186, respectively. When the point guess values are $\theta_0 = 150(50)500$ for true value θ. Then, using $\hat{\theta}$ for estimating θ, the corresponding ML estimates of δ_0, i.e. $\hat{\delta}_0 = \theta_0/\hat{\theta}$, are $0.47, 0.63, 0.79, 0.95, 1.11, 1.27, 1.43$. Table 1 presents the risks of

Table 1: The values of risk of shrinkage testimator $\hat{\theta}_{st}$ for $\alpha = 0.05, \gamma = 1, 5$ and selected values of $k = 0.1(0.3)0.9$.

		k				
$(\theta_0)\delta_0$	γ	0.1	0.3	0.5	0.7	0.9
(150)0.47	1	0.0933	0.0825	0.0735	0.0663	0.0605
	5	0.3357	0.3070	0.2787	0.2522	0.2289
(200)0.63	1	0.0763	0.06705	0.0611	0.0580	0.0576
	5	0.3141	0.2755	0.2465	0.2280	0.2195
(250)0.79	1	0.0382	0.0369	0.0392	0.0447	0.0530
	5	0.1531	0.1455	0.1528	0.1727	0.2017
(300)0.95	1	0.0163	0.0197	0.0265	0.0366	0.0502
	5	0.0484	0.0649	0.0964	0.1398	0.1912
(350)1.11	1	0.0206	0.0219	0.0269	0.0359	0.0495
	5	0.0692	0.0749	0.0970	0.1354	0.1882
(400)1.27	1	0.0442	0.0386	0.0372	0.0408	0.0506
	5	0.1776	0.1497	0.1396	0.1524	0.1906
(450)1.43	1	0.0777	0.0632	0.037	0.0492	0.053
	5	0.3167	0.2547	0.2071	0.1849	0.1979

the shrinkage testimator $\hat{\theta}_{st}$ for $\alpha = 0.05, \gamma = 1, 5$ and selected values of $k = 0.1(0.3)0.9$. From Table 1, the following points can be observed:

- When $\theta_0 = 150$ (a lower point guess) and $\gamma = 1$, the MLE performs well with respect to the shrinkage testimator. However, in this case, a shrinkage testimator with $k = 0.9$ is well with respect to the other shrinkage testimators. A similar pattern is observed when $\gamma = 5$ and $\theta_0 = 150, 200$.

- For the initial guess values near to the $\hat{\theta}$, i.e. $\delta_0 = 250, 250, 300, 350, 400$, the shrinkage testimators are better than $\hat{\theta}$.

- Selecting $\gamma = 1$, we see that the shrinkage testimators work better than those testimators when $\gamma = 5$. When θ_0 is significantly close, as in $\theta_0 = 300$, the shrinkage testimator with $k = 0.1$ is recommended for fixed values of γ. Also, a shrinkage testimator with $k = 0.1$ and $\gamma = 1$ is preferable.

Now, consider the estimation of θ when the guess value is $\theta_0 = 300$. The MLE of δ_0 is $\hat{\delta}_0 = \frac{\theta_0}{\hat{\theta}} = 0.95$. The value of the shrinkage factor k_1 founded by minimizing the risk of the shrinkage estimator $\hat{\theta}_s$ given in (1.1) is $k_1 = 0.0188$. The LRT statistic for testing the null hypothesis $H_0: \ \theta = 300$ is $\chi^2 = 16.67$. If we consider $\alpha = 0.05$, then the values of the lower and upper 25% points of a chi-square distribution with $2r = 16$ degrees of freedom are $q_1 = 6.91$ and $q_2 = 28.85$, respectively. This implies that the null hypothesis is accepted at the level 0.05. From Table 1, we observe that the shrinkage testimator with $k = 0.1$, say $\hat{\theta}_{st}^{0.1}$, has good performance. The estimates of $\hat{\theta}$, $\hat{\theta}_{st}^{0.1}$ and $\hat{\theta}_{st1}$ (the shrinkage testimator corresponding to k_1 in (3.5)) and their risks are summarized in Table 2 for $\gamma = 1, 5$. We observe from Table 2 that the shrinkage testimators $\hat{\theta}_{st1}$ and $\hat{\theta}_{st}^{0.1}$ are better than $\hat{\theta}$ and, overally, the shrinkage testimator $\hat{\theta}_{st}^{0.1}$ is recommended.

Table 2: The estimates of $\hat{\theta}$, $\hat{\theta}_{st}^{0.1}$ and $\hat{\theta}_{st1}$ and their risks (in parenthesis) for $\gamma = 1, 5$ in our example.

γ	$\hat{\theta}$	$\hat{\theta}_{st}^{0.1}$	$\hat{\theta}_{st1}$
1	312.7356(0.0581)	301.2736(0.0163)	300.2394(0.0165)
5	312.7356(0.2186)	301.2736(0.0484)	300.2394(0.0493)

5 Concluding remarks

In this paper, the problem of the shrinkage testimation for the scale-parameter of a Rayleigh distribution based on censored data under the RGL function is considered. The minimum risk estimator is derived and its risk is computed. A shrinkage testimator based on the closeness of the guess value and the true value is constructed. A comparison between these estimators and the minimum risk estimator are performed via calculation of relative efficiency of them. A lower level of significance, i.e. $\alpha = 0.01$, is recommended for various values of r. In particular, when $\gamma = 1$, the shrinkage testimators at $\alpha = 0.01$ are well.

The results given in the preceding sections for the Rayleigh distribution can also be extended to other distributions. Suppose that the type-II right censored component failure times $\mathbf{Y} = (Y_{1:n}, \cdots, Y_{r:n})$ are available, where Y is the failure time of a component with c.d.f. $F_Y(y; \theta)$ where θ is unknown.

- If Y is an exponential random variable with the p.d.f.

$$f(y; \theta) = \theta^{-1} \exp\{-y/\theta\}, \quad y > 0, \theta > 0,$$

then, $X = \sqrt{2Y} \sim Ray(\theta)$.

- If Y is a Weibull random variable with the p.d.f.

$$f(y; \nu, \theta) = \nu\theta^{-1} y^{\nu-1} \exp\{-y^{\nu}/\theta\}, \quad y > 0, \theta > 0,$$

where $\nu > 0$ is known, then $X = \sqrt{2Y^{\nu}} \sim Ray(\theta)$.

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